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Author for correspondence:
John Milton
e-mail: jmilton@kecksci.claremont.edu

Acting together, destabilizing influences can stabilize human balance

John Milton[1] and Tamas Insperger[2]

[1]W. M. Keck Science Center, The Claremont Colleges, Claremont, CA 91711, USA
[2]Department of Applied Mechanics, Budapest University of Technology, and MTA-BME Lendület Human Balancing Research Group, 1111 Budapest, Hungary

JM, 0000-0001-5037-2548; TI, 0000-0001-7518-9774

The causes of falling in the elderly are multi-factorial. Three factors that influence balance stability are the time delay, a sensory dead zone and the maximum ankle torque that can be generated by muscular contraction. Here, the effects of these contributions are evaluated in the context of a model of an inverted pendulum stabilized by time-delayed proportional–derivative (PD) feedback. The effect of the sensory dead zone is to produce a hybrid type of control in which the PD feedback is switched ON or OFF depending on whether or not the controlled variable is larger or smaller than the detection threshold, Π. It is shown that, as Π increases, the region in the plane of control parameters where the balance time (BT) is greater than 60 s is increased slightly. However, when maximum ankle torque is also limited, there is a dramatic increase in the parameter region associated with BTs greater than 60 s. This increase is due to the effects of a torque limitation on over-control associated with bang-bang type switching controllers. These observations show that acting together influences, which are typically thought to destabilize balance, can actually stabilize balance.

This article is part of the theme issue 'Nonlinear dynamics of delay systems'.

1. Introduction

Falls are associated with a high mortality and morbidity in the elderly. The rapidly growing elderly population makes it imperative to develop strategies to prevent falls, otherwise Western societies will soon be overcome by an

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'epidemic of falling'. Several studies have demonstrated that elderly patients with a history of falling have an increased variance in the fluctuations in their centre of pressure (COP) measured using a force platform during quiet standing [1–4]. Analysis of the properties of these fluctuations has shown promise of being able to identify subjects who are at increased risk of falling [5,6]. A variety of explanations have been offered to explain this increased variance, including an increase in the time delay for corrective movements, an increased sensory dead zone, decreased ankle muscle torque and changes in body posture. These observations, in turn, have sparked interest in a variety of therapeutic manoeuvres such as shoes with vibrating insoles to decrease the effect of an increased sensory dead zone [7] and exercise to strengthen lower leg muscles [8–10]. Issues related to increasing compliance have been addressed by incorporating exercises into activities that people like to do such as Tai Chi [11], dancing [12], swimming [13] and golf [14,15]. Despite all of these efforts, there has been only modest success in reducing the risk of falling in the elderly (e.g. [9,13]).

The observation that the upright position is intrinsically unstable places constraints on the nature of mechanisms that maintain balance. Typically, the destabilizing effects of time delays, sensory dead zone and limitations in ankle muscle torque on postural balance are considered independently. However, an elderly person probably has issues related to all of these parameters at the same time. Here, we show that, acting together, it is possible that these destabilizing factors can actually stabilize balance. This is because, as more conditions are added to the control law, balancing performance changes as a result of interactions between strong, small-scale nonlinearities (see also [16] and Case D in [17]). Failure to recognize the possibility that these factors do not necessarily affect balance in an additive manner is a major confound for interpreting the results of clinical trials designed to lower the risk of falling in the elderly.

Our discussion is organized as follows. In §2, we briefly review the properties of the fluctuations in COP during quiet standing and the role of delay differential equations for the investigation of postural stability. In particular, postural control is examined from the perspective of the role of time-delayed feedback in the stabilization of an inverted pendulum. In §3, we introduce a simplified model for standing that includes the effects of time delay, sensory dead zone and ankle muscle torque limitation. In §4, we show that each of these effects acting on their own can destabilize the upright position when the feedback is time sampled. However, acting together, these effects dramatically increase the range of the values of the control gains for which stability is possible.

2. Background

In a quiet room with eyes closed, the COP oscillations are of the order of 0.5° for healthy individuals with no history of falling. The bandwidth of the fluctuations is 0–3 Hz with a mean frequency of 0.9–1.3 Hz [18–20].

The complex nature of the fluctuations in COP has attracted a great deal of interest. There have been three lines of investigation. One possibility is that the fluctuations represent the dynamics of a stochastic time-delayed dynamical system [21–23]. A second possibility is that the dynamics represent a sampled feedback control system [24,25]. This possibility is supported by the experimental observation demonstrating that corrective movements for balance control are exerted by the nervous system intermittently [18,26]. This observation has also been made for stick balancing on the fingertip [27] and other human tracking tasks [28]. Possible mechanisms for generating a sampled data system include: (1) central control mechanisms related to the central refractory time [18,24]; (2) the presence of a sensory dead zone to produce a switching-type feedback [22,29]; and (3) a time-delayed dynamical system tuned at the edge of stability [25,27].

A third possibility is based on the observation that fluctuations in COP in either the anterior–posterior direction or the medial–lateral direction can, at times, appear to contain two components [30,31]: a 'slow' non-oscillatory component and a 'fast' oscillatory component. The 'slow' component corresponds to an exponential decay back to equilibrium that is perturbed

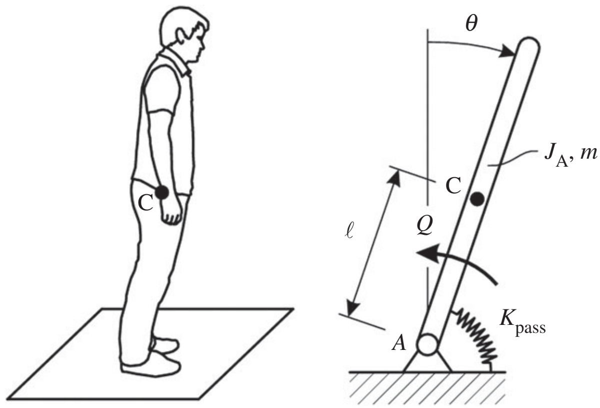

Figure 1. Pinned pendulum model for postural balance during quiet standing with eyes closed. The • indicates the position of the centre of mass. For definition of symbols, see §3.

stochastically. The 'fast' component is related to the time-delayed feedback control involved in the stabilization of an inverted pendulum. It has been suggested that the 'slow' component is present inside the feedback loop of the 'fast' component.

These three possibilities are not mutually exclusive. The common feature of each of them is an inverted pendulum stabilized by state-dependent and time-delayed feedback [32–37] (figure 1). The governing equation takes the form

$$\ddot{\theta}(t) - \omega_n^2 \theta(t) = f(t), \tag{2.1}$$

where θ is the vertical displacement angle, ω_n is the natural angular frequency of the system hung upside down and $f(t)$ is the control torque. A great number of possible choices of f have been considered, including state-dependent control (control depends on the state variables θ, $\dot{\theta}$, $\ddot{\theta}$) [32–35,37,38], predictor feedback (control depends on an internal model) [25,39,40], intermittent control [24], 'act-and-wait' control [41], noise-aided control [27] and nonlinear control strategies [16,37] (for a review see [42]).

All time-invariant nonlinear feedback controllers that can be written in the form $f(t) = g(\theta(t-\tau), \dot{\theta}(t-\tau))$ can be reduced to proportional–derivative (PD) feedback after linearization if the function g is smooth in both of its arguments. Thus, we limit our discussion to delayed PD types of feedback, namely, the feedback takes the form

$$f(t) = -k_p \theta(t-\tau) - k_d \dot{\theta}(t-\tau), \tag{2.2}$$

where k_p and k_d are the proportional and the derivative control gains and τ is the reaction time delay.

Figure 1 shows a 'pinned' inverted pendulum controlled by time-delayed PD feedback. The term 'pinned' refers to the fact that the position of the pivot point A of the inverted pendulum is fixed and hence the only possible motions are those confined to the anterior–posterior plane about the pivot. The fixed point of (2.1) with $f(t) = 0$ is a saddle. In order to analyse linear stability when $f(t) \neq 0$, it is useful to note that the period of the small-amplitude oscillations that occur when the pendulum is hung downwards is $T_p = 2\pi/\omega_n$ [38].

We focus on three causes of instability for human balance during quiet standing.

(a) Time delay

The observation that an inverted pendulum can be stabilized by time-delayed feedback draws attention to the importance of the inter-relationship between τ and ω_n. This relationship is summarized by the stabilizability parameter referred to as the critical delay, τ_{crit}. An inverted pendulum with natural angular frequency ω_n cannot be stabilized unless τ is less than a critical delay:

$$\tau \leq \tau_{crit} = \frac{\sqrt{2}}{\omega_n} = \frac{T_p}{\pi\sqrt{2}}, \tag{2.3}$$

i.e. T_p is divided by the two 'most popular' irrational numbers, π and $\sqrt{2}$ [38].

(b) Sensory dead zone

Irrespective of the choice of f, the control is ultimately influenced by the limitations of the nervous system to measure the angular position $\theta(t)$ and the angular velocity $\dot{\theta}(t)$. This is because all sensory receptors have a dead zone, namely a threshold below which changes in input are not reflected by changes in output [43,44]. Negative feedback controllers are designed to minimize $\theta(t)$. The presence of a finite dead zone means that $\theta(t)$ cannot be made arbitrarily small. This is a serious problem in situations in which the fluctuations to be controlled are of the same order of magnitude as the detection thresholds of sensory receptors. Indeed, for healthy individuals, the measured sensory dead zone is ≈20% of the magnitude of the fluctuations in COP (see §3).

The mathematical consequence of the presence of a sensory dead zone is to introduce a strong, local nonlinearity into the governing equations [45,46]. In particular, the feedback switches between ON and OFF depending on whether the controlled variable is larger or smaller than a threshold value. Consequently, (2.1) becomes, for example,

$$\ddot{\theta}(t) - \omega_n^2\theta(t) = \begin{cases} 0 & \text{if } |\theta(t-\tau)| < \Pi, \\ f(t) & \text{if } |\theta(t-\tau)| \geq \Pi, \end{cases} \tag{2.4}$$

where Π is the sensory threshold. In the engineering literature, such controllers are referred to as *hybrid* controllers [47,48]. Hybrid controllers are optimal when the output is bounded [47] and in the presence of stochastic perturbations minimize the effects of 'over-control' [22]. Here, we consider only the situation in which the switching criterion depends on the delayed value of the state variable, $\theta(t-\tau)$, in order to account for the reaction time delay; the case in which the switching condition depends on $\theta(t)$ in considered in [46]. The effect of feedback quantization is to generate limit cycle oscillations whose amplitude is a function of Π [17]. Indeed, a stable fixed point for (2.4) does not exist. When Π is sufficiently small, the amplitude of the oscillations can be small enough to practically approximate the dynamics of a stable fixed point. If, in addition to feedback quantization, the control signals are digitally sampled, it is possible that small-scale chaotic dynamics can arise, which are referred to as *microchaos* [29,49–52].

(c) Ankle torque saturation

An early hypothesis was that balance during quiet standing was maintained solely by ankle stiffness [19]. However, it was subsequently shown that, although ankle stiffness is a passive contributor to balance control during quiet standing, it is not sufficient to control balance without assistance from active neural feedback [53,54]. Falling of the elderly is often associated with the saturation of the control torque, i.e. the subjects cannot exert the required amount of torque at their ankle (see, for example, [8,55]). From a dynamical point of view, control torque saturation presents strong nonlinearity associated with subcritical bifurcation, which reduces the domain of attraction of the stabilized fixed point [56].

3. Model

Our model for postural stability during quiet standing takes the form

$$J_A\ddot{\theta}(t) - (mg\ell - K_{pass})\theta(t) = Q(t), \tag{3.1}$$

where J_A is the moment of inertia of the body with respect to the normal line via the pivot point A, m is the mass of the body, $g = 9.81\,\mathrm{m\,s^{-2}}$ is the gravitational acceleration, ℓ is the distance between the centre of gravity C and the suspension point A, and K_{pass} indicates the passive stiffness that is insufficient to resist falling against gravity [53]. Following [16], we set $m = 60\,\mathrm{kg}$, $J_A = 60\,\mathrm{kg\,m^2}$, $\ell = 1\,\mathrm{m}$ and $K_{pass} = 0.8\,mg\ell$. In order to account for the effects of control torque saturation and sensory thresholds for the different sensory inputs on balance control, the control torque is written in the form

$$Q(t) = \begin{cases} Q_{max} & \text{if } Q_p(t) + Q_d(t) \geq Q_{max}, \\ Q_p(t) + Q_d(t) & \text{if } |Q_p(t) + Q_d(t)| < Q_{max}, \\ -Q_{max} & \text{if } Q_p(t) + Q_d(t) \leq -Q_{max}, \end{cases} \tag{3.2}$$

where Q_{max} is the maximum exertable control torque and $Q_p(t)$ and $Q_d(t)$ are the components of the ankle torque related to the angular position and to the angular velocity. In the case of time-delayed PD feedback,

$$Q_p(t) = \begin{cases} 0 & \text{if } |\theta(t-\tau)| < \Pi_{pos}, \\ -K_p\theta(t-\tau) & \text{if } |\theta(t-\tau)| \geq \Pi_{pos}, \end{cases} \tag{3.3}$$

and

$$Q_d(t) = \begin{cases} 0 & \text{if } |\dot{\theta}(t-\tau)| < \Pi_{vel}, \\ -K_d\dot{\theta}(t-\tau) & \text{if } |\dot{\theta}(t-\tau)| \geq \Pi_{vel}, \end{cases} \tag{3.4}$$

with K_p and K_d being the proportional and the derivative control gains, respectively. Here, Π_{pos} and Π_{vel} are the sensory dead zones for the body's angular position and angular velocity, respectively. The sensory dead zone for the body's angular position, Π_{pos}, is $\approx 0.1^\circ$ for healthy adults with no balance problems. This dead zone has been estimated using small mechanical ankle displacements [57] and from the two-point correlation function [21,43]. Presumably, dead zones also exist for the detection of the angular velocity, Π_{vel}; however, these have not yet been measured. We assume that for healthy adults $\Pi_{vel} = 1\,\mathrm{deg\,s^{-1}}$. The value of the maximum control torque is taken to be $Q_{max} = 20\,\mathrm{N\,m}$ [53]. Estimates for reaction delay τ for postural sway are 90–125 ms for healthy subjects [30,58–60] and these estimates are increased by less than 50% in the elderly [4,61]. We varied τ from 100 ms to 250 ms. For these parameters, the oscillations in the postural sway angle are of the order of 0.5° as observed experimentally for large ranges of choices of pairs (K_p, K_d) (see figures below). When $\Pi_{pos} = 0$ and $\Pi_{vel} = 0$, equation (3.1) with (3.2) and (3.3)–(3.4) give (2.1) with (2.2), where

$$\omega_n = \sqrt{\frac{mg\ell - K_{pass}}{J_A}}, \quad k_p = \frac{K_p}{J_A} \quad \text{and} \quad k_d = \frac{K_d}{J_A}. \tag{3.5}$$

In order to take the sampled nature of the negative feedback into account for balance stability, we integrate (3.1) using the semi-discretization method [52,62] using integration time step $\Delta t = 10\,\mathrm{ms}$. Although Δt may be larger due to the effects of a central refractory time [18], we used 10 ms since this time step was used previously for human control of an inverted pendulum using an internal model [25,40]. This integration procedure also includes a zero-order hold and hence approximates the observation that the nervous system requires a finite time to plan a movement and cannot begin planning a new movement until the previous movement is completed [18,28]. Increasing Δt increases the microchaotic contribution to the solutions [29].

4. Results

Figure 2 shows the effects of, respectively, increasing τ, increasing Π_{pos} and Π_{vel}, and decreasing Q_{max} on postural stability for $K_p = 634\,\mathrm{N\,m\,rad^{-1}}$ and $K_d = 288\,\mathrm{N\,m\,s\,rad^{-1}}$. In all cases, the effect is to destabilize the upright position.

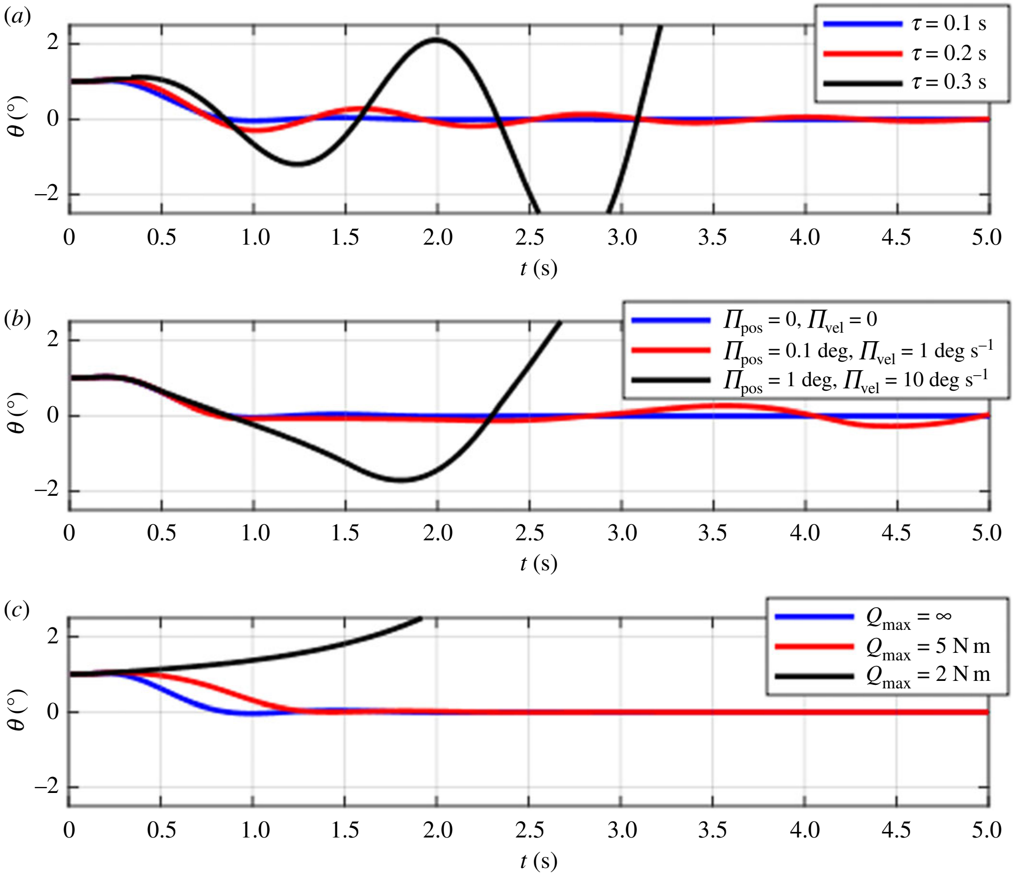

Figure 2. Effect of increasing reaction time τ (*a*), increasing sensory dead zones Π_{pos} and Π_{vel} (*b*) and decreasing maximum control torque Q_{max} (*c*). (Online version in colour.)

An alternative way to investigate the effects of these parameters on balance stability is to estimate the region in the plane (K_p, K_d) where balance can be maintained. We define the balance time (BT) as the time instant when θ exceeds a limit value θ_{lim}, which is the angle where the centre of mass gets outside of the basin of support of the feet. Assuming a foot length of 28 cm with $\ell = 1$ m, then $\theta_{lim} \approx 8°$. When θ does not exceed θ_{lim} for $t_{max} = 60$ s, then the simulations were terminated and BT was set to 60 s.

The effect of changes in τ, Π_{pos}, K_{pass} and Q_{max} were investigated numerically. Series of numerical simulations were performed via the semi-discretization method [52,62] for a range of pairs (K_p, K_d). The initial conditions for the simulations were a steady-state tilted position $\theta = 1°$. Several other types of initial conditions were also tested and it was found that the effect of the choice on the initial conditions did not affect the final BT.

The linear stability analysis for (3.1) with time-delayed PD feedback is one of the most frequently cited examples in dynamics and control theory (see, for example, [62]). For a given τ, there is a D-shaped region of linear stability in the plane (K_p, K_d) (see region enclosed by yellow lines in figures 3 and 4). For choices of (K_p, K_d) within this region, the upright fixed point is stable. As τ increases, the D-shaped region of stability decreases in size (figure 3).

The effect of the introduction of a sensory dead zone according to (3.3)–(3.4) on the BT is also shown in figure 3. The region in the plane (K_p, K_d) for which BT > 60 s in the presence of a sensory dead zone (light blue line) is slightly larger than the linear stability region (yellow line) observed when $\Pi_{pos} = 0$ and $\Pi_{vel} = 0$. Thus, the effects of a dead zone are not necessarily destabilizing

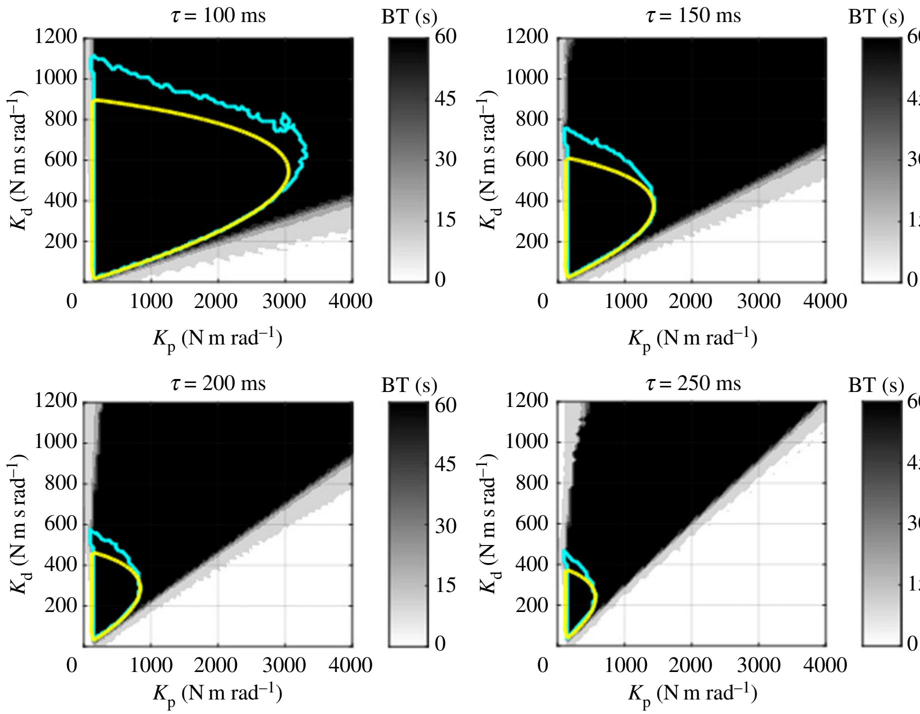

Figure 3. Balance time for different feedback delays with $\Pi_{pos} = 0.1$ deg, $\Pi_{vel} = 1$ deg s^{-1} and $Q_{max} = 20$ N m. The yellow line encloses the region of linear stability for (2.1). The light blue line shows the boundaries where the solution remains bounded for BT $= 60$ s with a dead zone, but without control torque saturation.

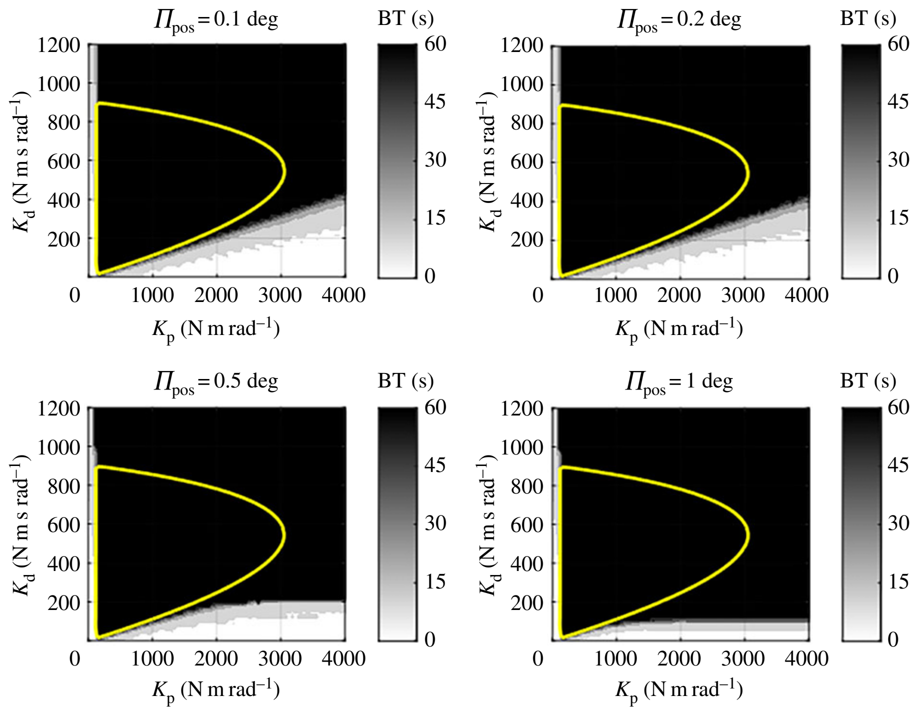

Figure 4. Balance time for different sensory thresholds for position perception with $\tau = 100$ ms, $\Pi_{vel} = 1$ deg s^{-1} and $Q_{max} = 20$ N m. The yellow line encloses the region of linear stability.

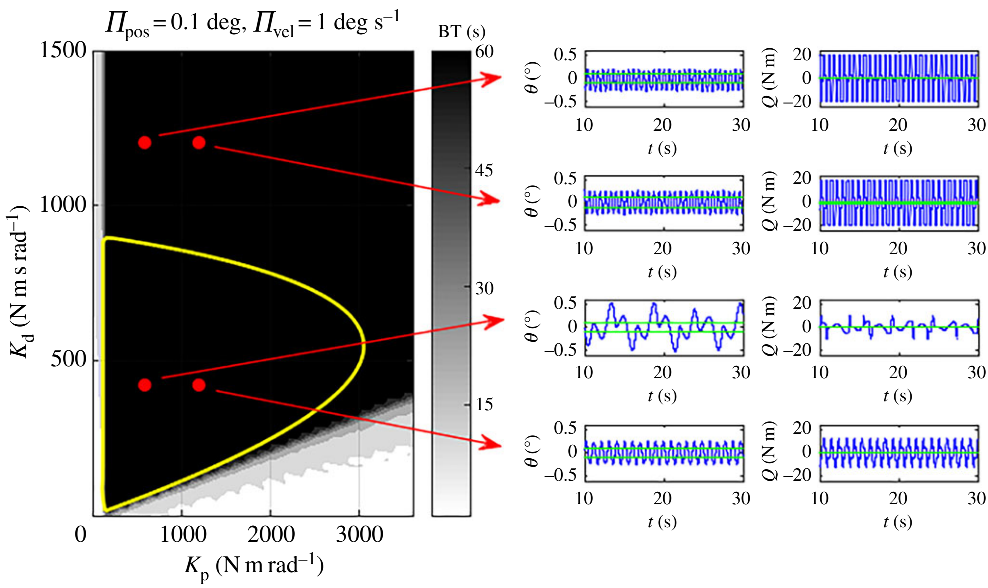

Figure 5. Simulations for θ and Q associated with different control gain pairs.

[17,47]. It is of interest to note that control engineers often introduce a dead zone near a desired equilibrium to save energy as well as to minimize wear and tear on an actuator [63].

The effect of the addition of a limitation on ankle muscle torque according to (3.2) is shown by the grey scale in figures 3 and 4. There is a large increase in the region in (K_p, K_d) for which BT > 60 s. In particular, this region is much greater than the region when $\Pi_{pos} = 0$ and $\Pi_{vel} = 0$ and there is no muscle torque limitation (yellow line). The benefit in control by limiting ankle muscle torque is an example of the effects of over-control. In time-delayed feedback control, a too large response by the controller to a given deviation can lead to destabilization [22,47]. Placing a limit on ankle muscle torque decreases the response triggered by a sensory threshold crossing and hence improves control. This explains the huge increase of the region of BT > 60 s.

Numerical simulations show that the region of BT > 60 s does not significantly change with the decrease of Q_{max}. The effect of the delay τ and the sensory threshold Π_{pos} is more pronounced, as shown in figures 3 and 4.

Figure 5 compares the time series generated by (3.1) with (3.2) and (3.3)–(3.4) for different choices of (K_p, K_d) in the presence of a sensory dead zone and a muscle torque limitation. The saturation of the torque allows larger (K_p, K_d) pairs. For large control gains, this concept leads to a 'bang-bang' control, i.e. the control torque is switching between $-Q_{max}$ and Q_{max}, similarly to the scalar model in [45]. It should be noted that, as the values of (K_p, K_d) approach the lower stability boundary, the frequency of the oscillations approaches that observed for postural sway, namely 0.9–1.3 Hz. This region of the plane (K_p, K_d) has also been shown to be more robust against uncertainties in the parameters of the mechanical model [25,64]. For choices of (K_p, K_d) between these extremes, complex solutions exist (see, for example, the third row of panels on the right of figure 5).

5. Discussion

The occurrence of a fall in an elderly person is typically a rare event, say 1–2 falls per year. The risk factors for a fall are of two types. First, there can be changes in the parameters related to the stability of the balance control mechanisms. Here, we have emphasized the destabilizing roles of increases in τ and Π_{pos} and decreases in Q_{max}. The important advantage of studies of

COP fluctuations for elderly subjects during quiet standing is that these measurements are non-invasive and can be made under conditions where the risk of falling is minimal. Abnormalities in COP do seem to be correlated with increased risk of falling [5,6]. Second, there can be problems related to the subject's ability to anticipate and react to sudden and unexpected threats to balance. For example, a subject's ability to maintain balance during activities of daily living can be diminished by the side effects of medications and co-morbidities such as diabetes and cerebrovascular disease [65,66].

Intuitively one might expect that the more risk factors that a subject has, the greater the number of falls they experience. However, no such simple relationship is observed. Our investigations provide insight into the non-intuitive relationship between risk factors and number of falls. In particular, we identify an important confound, namely, different destabilizing influences for balance control can interact to stabilize the upright position. In other words, human balance is controlled by a system that is robust to abnormalities in its component parts. However, the quest for identifying abnormalities in COP fluctuations is not completely hopeless. Indeed, a wide range of different dynamical behaviours exists ranging from simple oscillations with a frequency characteristic of postural sway for healthy individuals to more complex types of rhythms to bang-bang type solutions. It is possible that not all of these solutions are stable when the properties of the ageing musculoskeletal system are taken into consideration. Thus it is possible that techniques that examine, for example, changes in the qualitative nature of postural sway dynamics may be useful (e.g. [6]).

6. Conclusion

The dynamics of an inverted pendulum stabilized with time-delayed PD feedback change when a sensory dead zone and a limitation of ankle muscle torque are included. In the absence of a sensory dead zone and no limitation on ankle torque, there can be a stable fixed point. The addition of the sensory dead zone replaces the fixed point with a limit cycle. The BT is increased further by a limitation on muscle torque because this reduces the possibility of over-control. As the muscle torque is weakened, the control more and more resembles 'bang-bang' type feedback. The combination of a sensory dead zone and a limitation on ankle muscle torque greatly increases the region in parameter space where the BT is greater than 60 s. Thus, acting together, these contributions to control are not necessarily destabilizing.

Data accessibility. This article has no additional data.

Authors' contributions. J.M. and T.I. conceived of and designed the study. T.I. performed the numerical simulations. J.M. drafted the manuscript. Both authors read and approved the manuscript.

Competing interests. The authors declare that they have no competing interests.

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References

1. Abrahamová D, Hlavačka F. 2008 Age-related changes of human balance during quiet stance. *Physiol. Res.* **57**, 957–964. See https://www.biomed.cas.cz/physiolres/pdf/57/57_957.pdf.
2. Bird M-L, Pittaway JK, Cuisick I, Rathray M, Ahuja KDK. 2013 Age-related changes in physical fall risk factors: results from a 3 year follow-up of community dwelling older adults in Tasmania, Australia. *Int. J. Environ. Res. Public Health* **10**, 5989–5997. (doi:10.3390/ijerph10115989)
3. Melzer I, Benjuya N, Kaplanski J. 2004 Postural stability in the elderly: a comparison between fallers and non-fallers. *Age Aging* **33**, 602–607. (doi:10.1093/ageing/afh218)

4. Woollacott MH, Shumway-Cook A, Nahser LM. 1986 Aging and postural control: changes in sensory organization and muscular coordination. *Int. J. Aging Human Dev.* **23**, 97–144. (doi:10.2190/VXN3-N3RT-54JB-X16X)
5. Johansson J, Nordström A, Gustafson Y, Westling C, Nordtröm P. 2017 Increased postural sway during quiet stand as a risk factor for prospective falls in community-dwelling elderly individuals. *Age Ageing* **46**, 964–970. (doi:10.1093/ageing/afx083)
6. Zhou J, Habtemariam D, Iloputaife I, Lipsitz LA, Manor B. 2017 The complexity of standing postural sway associates with future falls in community-dwelling older adults: the MOBILIZE Boston study. *Sci. Rep.* **7**, 2924. (doi:10.1038/s41598-017-03422-4)
7. Lipsitz L, Lough M, Niemi J, Travison T, Howlett H, Manor B. 2015 A shoe insole delivery subsensory vibration noise improves balance and gait in healthy elderly people. *Arch. Phys. Med. Rehabil.* **96**, 432–439. (doi:10.1016/j.apmr.2014.10.004)
8. Choi J-H, Kim N-J. 2015 The effects of balance training and ankle training on the gait of elderly people who have fallen. *J. Phys. Ther. Sci.* **27**, 139–142. (doi:10.1589/jpts.27.139)
9. Crilly RG, Williams DA, Trenholm KJ, Hayes KC, Delaquerrière-Richardson LFO. 1989 Effect of exercise on postural sway in the elderly. *Gerontology* **35**, 137–143. (doi:10.1159/000213012)
10. Ishigaki EY, Ramos LG, Carvalho ES, Lunardi AC. 2014 Effectivness of muscle strengthening and description of protocols for preventing falls in the elderly: a systematic review. *Brazil. J. Phys. Ther.* **18**, 111–118. (doi:10.1590/S1413-35552012005000148)
11. Huang ZG, Feng YH, Li YH, Lv CS. 2017 Systematic review and meta-analysis: Tai-Chi for preventing falls in older adults. *BMJ Open* **7**, e013661. (doi:10.1136/bmjopen-2016-013661)
12. Franco MR, Sherrington C, Tiedemann A, Pereira LS, Perscini MR, Faria CRS, Pinto RZ, Pastre CM. 2016 Effectiveness of senior dance on risk factors for falls in older adults (DanSE): a study protocol for a randomized controlled trial. *BMJ Open* **6**, e013995. (doi:10.1136/bmjopen-2016-013995)
13. Merom D, Stanaway FF, Handelsman DJ, Waite LM, Siebel MJ, Blyth FM, Naganathan V, Cumming RG. 2014 Swimming and other sporting activities and the rate of falls in older men: longitudinal findings from the Concord Health and Aging in Men Project. *Am. J. Epidemiol.* **180**, 830–837. (doi:10.1093/aje/kwu199)
14. Chow DHK, Mann SKF. 2015 Effect of cyber-golfing on balance amongst the elderly in Hong Kong: a pilot randomized trial. *Hong Kong J. Occup. Ther.* **26**, 9–13. (doi:10.1016/j.hkjot.2015.08.001)
15. Tsang WW, Hui-Chan CW. 2004 Effects of exercise on joint sense and balance in elderly men: Tai Chi versus golf. *Med. Sci. Sports Exerc.* **36**, 658–667. (doi:10.1249/01.MSS.87090.2E)
16. Asai Y, Tasaka Y, Nomura K, Casidio M, Morasso P. 2009 A model of postural control in quiet standing: robust compensation of delay-induced instability using intermittent activation of feedback control. *PLoS ONE* **4**, e6169. (doi:10.1371/journal.pone.0006169)
17. Stepan G, Milton J, Insperger T. 2017 Quantization improves stabilization of dynamical systems with delayed feedback. *Chaos* **27**, 114306. (doi:10.1063/1.5006777)
18. Loram ID, Maganaris CN, Lakie M. 2005 Human postural sway results from frequent, ballistic bias impulses by soleus and gastrocnemius. *J. Physiol.* **564**, 295–311. (doi:10.1113/jphysiol.2004.076307)
19. Winter DA, Patla AE, Prince F, Ishac MG, Gielo-Perczak K. 1998 Stiffness control of balance in quiet standing. *J. Neurophysiol.* **80**, 1211–1221. (doi:10.1152/jn.1998.80.3.1211)
20. Yamamoto T, Smith CE, Suzuki Y, Kiyono K, Tanahashi T, Sakuda S, Morasso P, Nomura T. 2015 Universal and individual characteristics of postural sway during quiet standing in healthy young adults. *Phys. Rep.* **3**, e12329. (doi:10.14814/phy2.12329)
21. Collins JJ, De Luca CJ. 1994 Random walking during quiet standing. *Phys. Rev. Lett.* **73**, 764–767. (doi:10.1103/PhysRevLett.73.764)
22. Milton JG, Cabrera JL, Ohira T. 2008 Unstable dynamical systems: delays, noise and control. *Europhys. Lett.* **83**, 48001. (doi:10.1209/0295-5075/83/48001)
23. Newell KM, Slobounov SM, Slobounova ES, Molenaar PCM. 1997 Stochastic processes in postural center-of-pressure profiles. *Exp. Brain Res.* **113**, 158–164. (doi:10.1007/BF02454152)
24. Gawthrop P, Loram I, Gollee H, Lakie M. 2014 Intermittent control models of human standing: similarities and differences. *Biol. Cybern.* **108**, 159–168. (doi:10.1007/s00422-014-0587-5)

25. Milton J, Meyer R, Zhvanetsky M, Ridge S, Insperger T. 2016 Control at stability's edge minimizes energetic costs: expert stick balancing. *J. R. Soc. Interface* **13**, 20160212. (doi:10.1098/rsif.2016.0212)
26. Loram ID, Lakie M. 2002 Human balancing of an inverted pendulum: position control by small, ballistic-like, throw and catch movements. *J. Physiol.* **540**, 1111–1124. (doi:10.1113/jphysiol.2001.013077)
27. Cabrera JL, Milton JG. 2002 On–off intermittency in a human balancing task. *Phys. Rev. Lett.* **90**, 158702. (doi:10.1103/PhysRevLett.89.158702)
28. Neilson PD, O'Dwyer NJ, Neilson MD. 1988 Internal models and intermittency: a theoretical account of human tracking behavior. *Biol. Cybern.* **58**, 101–112. (doi:10.1007/BF00364156)
29. Milton JG, Insperger T, Cook W, Harris DM, Stepan G. 2018 Microchaos in human postural balance: sensory dead zones and sampled time-delayed feedback. *Phys. Rev. E* **98**, 022223. (doi:10.1103/PhysRevE.98.022223)
30. Kiemel T, Oie KS, Jeka JJ. 2006 Slow dynamics of postural sway are in the feedback loop. *J. Neurophysiol.* **95**, 1410–1418. (doi:10.1152/jn.01144.2004)
31. Zatsiorsky VM, Duarte M. 2000 Rambling and trembling in quiet standing. *Motor Control* **4**, 185–200. (doi:10.1123/mcj.4.2.185)
32. Insperger T, Milton J. 2017 Stick balancing with feedback delay, sensory dead zone, acceleration and jerk limitation. *Procedia IUTAM* **22**, 59–66. (doi:10.1016/j.piutam.2017.08.009)
33. Insperger T, Milton J, Stepan G. 2013 Acceleration feedback improves balancing against reflex delay. *J. R. Soc. Interface* **36**, 2156–2163. (doi:10.1098/rsif.2012.0763)
34. Lockhart DB, Ting LH. 2007 Optimal feedback transformation for balance. *Nat. Neurosci.* **10**, 1329–1336. (doi:10.1038/nn1986)
35. Maurer CV, Peterka RJ. 2005 A new interpretation of spontaneous sway measures based on a simple model of human postural control. *J. Neurophysiol.* **93**, 189–200. (doi:10.1152/jn.00221.2004)
36. Milton J, Cabrera JL, Ohira T, Tajima S, Tonosaki Y, Eurich CW, Campbell SA. 2009 The time-delayed inverted pendulum: implications for human balance control. *Chaos* **19**, 026110. (doi:10.1063/1.3141429)
37. Nomura T, Oshikawa S, Suzuki Y, Kiyone K, Morasso P. 2013 Modeling human postural sway using an intermittent control and hemodynamic perturbations. *Math. Biosci.* **245**, 86–95. (doi:10.1016/j.mbs.2013.02.002)
38. Stepan G. 2009 Delay effects in the human sensory system during balancing. *Phil. Trans. R. Soc. A* **367**, 1195–1212. (doi:10.1098/rsta.2008.0278)
39. Krstic M. 2009 *Delay compensation for nonlinear, adaptive, and PDE systems*. Boston, MA: Birkhäuser.
40. Mehta B, Schaal S. 2002 Forward models in visuomotor control. *J. Neurophysiol.* **88**, 942–953. (doi:10.1152/jn.2002.88.2.942)
41. Insperger T. 2006 Act and wait concept for time-continuous control systems with feedback delay. *IEEE Trans. Control Syst. Technol.* **14**, 974–977. (doi:10.1109/TCST.2006.876938)
42. Insperger T, Milton J. 2014 Sensory uncertainty and stick balancing at the fingertip. *Biol. Cybern.* **108**, 85–101. (doi:10.1007/s00422-013-0582-2)
43. Milton J, Insperger T, Stepan G. 2015 Human balance control: dead zones, intermittency and microchaos. In *Mathematical approaches to biological systems: Networks, oscillations and collective phenomena* (eds T Ohira, T Ozawa), pp. 1–28. New York, NY: Springer.
44. Northrup RB. 2001 *Introduction to dynamic modeling of neuro-sensory systems*. Boca Raton, FL: CRC Press.
45. Eurich CW, Milton JG. 1996 Noise-induced transitions in human postural sway. *Phys. Rev. E* **54**, 6681–6684. (doi:10.1103/PhysRevE.54.6681)
46. Kowalczyk P, Glendinning P, Brown M, Medrano-Cerda G, Dallali H, Shapiro J. 2012 Modeling human balance using switched systems with linear feedback control. *J. R. Soc. Interface* **9**, 234–245. (doi:10.1098/rsif.2011.0212)
47. Flügge-Lotz I. 1968 *Discontinuous and optimal control*. New York, NY: McGraw-Hill.
48. Guckenheimer J. 1995 A robust hybrid stabilization strategy for equilibria. *IEEE Trans. Automat. Control* **40**, 321–326. (doi:10.1109/9.341802)
49. Enikov E, Stepan G. 1998 Microchaotic motion of digitally controlled machines. *J. Vib. Control* **4**, 427–443. (doi:10.1177/107754639800400405)

50. Gyebroszki G, Csernak G. 2019 The hybrid micro-chaos map: digitally controlled inverted pendulum with dry friction. *Period. Polytech. Mech. Eng.* **63**, 148–155. (doi:10.3311/PPme.13300)
51. Haller G, Stepan G. 1996 Micro-chaos in digital control. *J. Nonlinear Sci.* **6**, 415–448. (doi:10.1007/BF02440161)
52. Insperger T, Milton J, Stepan G. 2015 Semi-discretization for time-delayed neural balance control. *SIAM J. Appl. Dyn. Syst.* **14**, 1258–1277. (doi:10.1137/140975632)
53. Loram ID, Lakie M. 2002a Direct measurement of human ankle stiffness during quiet standing: the intrinsic mechanical stiffness is insufficient for stability. *J. Physiol.* **545**, 1041–1053. (doi:10.1113/jphysiol.2002.025049)
54. Loram ID, Lakie M, Di Giulio I, Maganaris CN. 2009 The consequences of short-range stiffness and fluctuating muscle activity for proprioception of postural joint rotations: the relevance to human standing. *J. Neurophysiol.* **102**, 460–474. (doi:10.1152/jn.00007.2009)
55. Pijnappels M, Reeves ND, Maganaris CN, van Dieën J. 2008 Tripping without falling; lower limb strength, a limitation for balance recovery and a target for training in the elderly. *J. Electromyogr. Kinesiol.* **18**, 188–196. (doi:10.1016/j.jelekin.2007.06.004)
56. Zhang L, Stepan G, Insperger T. 2018 Saturation limits the contribution of acceleration feedback to balancing against reaction delay. *J. R. Soc. Interface* **15**, 20170771. (doi:10.1098/rsif.2017.0771)
57. Fitzpatrick R, McCloskey DI. 1994 Proprioceptive, visual and vestibular thresholds for the perception of sway during standing in humans. *J. Physiol.* **478**, 173–186. (doi:10.1113/jphysiol.1994.sp020240)
58. Li Y, Levine WS, Loeb GE. 2012 A two-joint human postural control model with realistic neural delays. *IEEE Trans. Neural Syst. Rehab. Eng.* **20**, 738–748. (doi:10.1109/TNSRE.2012.2199333)
59. Pasma JH, Boonstra TA, van Kordelan J, Spyropoulou VV, Schouten AC. 2017 A sensitivity analysis of an inverted pendulum balance control model. *Front. Comput. Neurosci.* **11**, 99. (doi:10.3389/fncom.2017.00099)
60. Woollacott MH, von Hosten C, Rösblad B. 1986 Relation between muscle response onset and body segmental movements during postural perturbations in humans. *Exp. Brain Res.* **72**, 593–604. (doi:10.1007/BF00250604)
61. Kanekar N, Aruin AS. 2014 The effect of aging on anticipatory postural control. *Exp. Brain Res.* **232**, 1127–1136. (doi:10.1007/s00221-014-3822-3)
62. Insperger T, Stepan G. 2011 *Semi-discretization for time-delay systems: stability and engineering applications*. New York, NY: Springer.
63. Wescott T. 2006 *Applied control theory for embedded systems*. Burlington, MA: Elsevier.
64. Hajdu D, Milton J, Insperger T. 2016 Extension of stability radius to neuromechanical systems with structured real perturbations. *IEEE Trans. Neural Syst. Rehab. Eng.* **24**, 1235–1242. (doi:10.1109/TNSRE.2016.2541083)
65. Sorond FA *et al.* 2011 Neurovascular coupling is impaired in slow walkers: the MOBILIZE Boston study. *Ann. Neurol.* **70**, 213–220. (doi:10.1002/ana.v70.2)
66. T'challa AE, Dufour AB, Travision TG, Habtemariam D, Ikoputaife I, Manor B, Lipsitz LA. 2011 Patterns, predictors, and outcomes of falls trajectories in older adults: the MOBILIZE Boston study with 5 years of follow-up. *PLoS ONE* **9**, e106363. (doi:10.1371/journal.pone.0106363)