## Reduction of ground-foot impact intensity of a hopping leg model on slopes

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Ground-foot impact intensity is the main source of injuries and energy loss during walking and running. The kinematic properties highly influence the intensity of the impact. Our recently developed one legged planar model is extended to obtain hopping motion with minimal impact intensity when the model moves on different slopes.

Our single-legged, planar mechanical model depicted in Fig. 1 is an extension of the model in [1], namely, the ground has an inclination of angle  $\alpha$  as opposed to the horizontal ground in the model in [1].

The overall model consists of the equation of motion and the control. Segments 1, 2 and 3 correspond to the foot, shank and thigh, respectively. Point A corresponds to the tiptoe, ideal joints B, C and D correspond to the ankle, the knee and the hip, respectively. The reaction wheel plays the role of the upper body: the torque  $M_D$  that rotates the thigh has the reaction torque exerted on the wheel. The wheel has mass  $m_r$  and moment of inertia  $J_r$ . The homogeneous, prismatic bars have masses  $m_i$  and lengths  $l_i$ , i = 1...3. The overall centre of gravity (CoG) is located at point G. The segments are interconnected by torsional springs with stiffnesses  $k_B$  and  $k_C$ . Actuating torques  $M_B$ ,  $M_C$  and  $M_D$  assist the motion according to the control, which will be introduced below.

The model has a total of 6 DoFs in the flight-phase:  $\mathbf{q}^{f} = [x_{A}, z_{A}, \theta_{1}, \theta_{12}, \theta_{23}, \theta_{r}]$  (where  $x_{A}$  and  $z_{A}$  are the Cartesian coordinates of the tiptoe); and it has 4 DoFs in ground-phase:  $\mathbf{q}^{g} = [\theta_{1}, \theta_{12}, \theta_{23}, \theta_{r}]$ . We assume that the ground-foot impact is completely inelastic, there is no rebound and the friction coefficient is high enough to prevent foot slip. Above assumptions allow us to constrain the tiptoe to the ground until the contact force is positive.



Fig. 1: Hopping leg model on variable inclination angle slope

Fig. 2: Schematic picture of a single period: flight-phase, ground-foot impact, ground-phase and ground foot detachment

The oscillations of the leg segments are suppressed by  $M_{\rm B}$  and  $M_{\rm C}$  in the flight-phase. A proportional-derivative controller realized by  $M_{\rm D}^{\rm f}$  drives the tiptoe (point A) in a specified horizontal position near to the centre of mass position  $x_{\rm G}$  in order to avoid falling over.

$$M_{\rm B}^{\rm f} = -D_{\rm B}\dot{\theta}_{12}, \quad M_{\rm C}^{\rm f} = -D_{\rm C}\dot{\theta}_{23}, \quad M_{\rm D}^{\rm f} = P(x_{\rm A} - (x_{\rm G} + x_{\Delta})) + D(\dot{x}_{\rm A} - \dot{x}_{\rm G}). \tag{1}$$

Term  $x_{\Delta} = P_{\Pi} \Pi_{A} - K_{\nu}$  modifies the desired tiptoe position regarding the angular momentum  $\Pi_{A}$  calculated for point A.  $K_{\nu}$  affects the horizontal locomotion speed. The constrained motion space kinetic energy (CMSKE) [2] is absorbed in each stride because of the ground-foot impact. CMSKE is recovered by means of the control torques  $M_{B}^{g}$  and  $M_{C}^{g}$  in the ground-phase. The total mechanical energy *E* is kept at the arbitrarily chosen energy level  $E_{0}$ .  $M_{D}^{g}$  prevents the continuous growth of the angular velocity  $\dot{\theta}_{r}$ .

$$M_{\rm B}^{\rm g} = P_E(E - E_0)\,{\rm sgn}(\dot{\theta}_{12})\,,\quad M_{\rm C}^{\rm g} = P_E(E - E_0)\,{\rm sgn}(-\dot{\theta}_{23})\,,\quad M_{\rm D}^{\rm g} = -P_{\rm r}\theta_{\rm r} - D_{\rm r}\dot{\theta}_{\rm r}\,.$$
(2)

We proved that stable periodic motion, i.e. hopping exists, in which flight and ground phase alternately interchanges each other as Fig. 2 shows. The flow function Jacobian  $\Phi$  for a total period was obtained as the composition of the flow Jacobians  $\Phi_f$  and  $\Phi_g$  for the continuous phases (calculated using the variational equation  $\dot{\Phi} = \nabla_x (\mathbf{f}(\mathbf{x})) \Phi$ ) and the Jacobians regarding the impulsive dynamics, i.e. impact and ground-foot detachment (for more details see: [3]).  $\Phi$  was used when the periodic solutions were found by shooting method and  $\Phi$  was also used for judging the stability.

Fig. 3 shows how the control parameters  $K_v$  and  $E_0$  affect the horizontal locomotion speed and the vertical amplitude of the hopping motion. Cases A, B, C and D are indicated here and the path of CoG and the tiptoe are plotted in Fig. 4 for each cases. CMSKE is an indicator of impact intensity [2]. Fig. 5 clearly indicates that the CMSKE denoted by  $T_c$  has a minimum. When impact intensity minimization is aimed, the optimal  $E_0$  value has to be chosen for a certain speed control parameter  $K_v$ . Figures 3-5 summarizes results for horizontal ground ( $\alpha = 0$ ).



Fig. 5: CMSKE normalized by the overall kinetic energy T

The above methodology is applied for inclined ground, when  $\alpha \neq 0$ . We report that inclination angle  $\alpha$  highly affects the ground-foot impact intensity and the optimal value of control parameter  $E_0$ . The results show that downhill locomotion ( $\alpha < 0$ ) induces more intensive ground-foot impacts than uphill hopping motion ( $\alpha < 0$ ). In future work the model will contain two legs and the control parameters will be tuned in such way that the generated motion will be comparable with laboratory experiment subjects' motion. Then the results will be applicable in practice; in artificial bipedal systems and in motion analysis of runners.

## References

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