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Stabilizability of mechanical systems subjected to digital PIDA control

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Abstract

This paper presents a study on the limits of stabilizability of unstable second-order dynamical systems by means of digital proportional-integral-derivative-acceleration (PIDA) feedback. Four different models are considered, which are all governed by the same dimensionless second-order differential equation. The mathematical model under analysis is a hybrid system involving terms with piecewise constant arguments due to the discrete sampling and actuation of the controller. Closed form formulas are derived for the domain of stability and for the limits of stabilizability as function of the system parameters, the sampling period and the control gains. It is concluded that while the acceleration term extends the limit of stabilizability, the integral term reduces stabilizability properties.

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Keywords: stabilizability, PIDA control, acceleration feedback, digital control, feedback control, zero-order hold, delay

1. Introduction

It is known for a long time that stabilization of an unstable equilibrium by means of delayed feedback is limited by the extent of the delay in the feedback loop. For a second-order system (e.g., an inverted pendulum) subjected to a continuous-time proportional-derivative (PD) feedback, the critical feedback delay limiting stabilizability can be given in closed form[1,2,3]. This critical delay can essentially be extended by involving the delayed acceleration into the feedback term[4,5], which gives a proportional-derivative-acceleration (PDA) feedback. It is also known that involving an integral term, thus applying a proportional-integral-derivative-acceleration (PIDA) feedback, does not further increase of the critical delay[6]. Stabilizability conditions for these continuous-time PD, PDA or PIDA feedback systems can be given by means of the method of D-subdivision and its generalizations[2,7,8].

In industrial applications, feedback mechanisms are implemented using digital controllers, which cannot be described as continuous-time dynamical systems any more, but rather can be represented as discrete-time maps[9,10,11,12,13]. Stabilizability issues still arise for digitally controlled machines, but here, the role of the feedback delay is taken by

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the sampling period. The critical size of the sampling period, which limits the stabilizability of a second-order system (e.g., an inverted pendulum) subjected to a digital PD feedback, can be given in closed form as function of the system parameters [14]. This criteria can be generalized for cases where additional delays, which are integer multiples of the sampling period, show up in the feedback loop [15]. Similarly to the continuous-time feedback systems, the critical sampling period is increased when the acceleration is involved in the feedback term (digital PDA controller) [16]. The goal of this paper is to analyze whether the limit of stabilizability can be further extended by the inclusion of an integral term (digital PIDA controller). For this analysis, closed-form formulas are derived which ensure the stability of the system, and the limits of stabilizability are also determined in the space of control parameters. During the analysis, the zero-order hold of the controller and the effect of sampling in the closed-loop system are considered using a sampling and actuation scheme according to Stepan [10].

In this paper, an unstable second-order system is analyzed, which is often associated with the model of the inverted pendulum. Actually, unstable second-order systems describes more general processes. When the goal of the feedback control system is to track a desired path or to simply remain in the proximity of a desired equilibrium position in the gravitational field, then the control action can usually be decomposed to a feedforward and a feedback term. The feedforward terms are determined by the inverse dynamics associated with the desired motion, while the role of the feedback term is to compensate for the error caused by the inaccurate inverse dynamics model. The variational system about the desired motion in this case is a second-order differential equation with a feedback term. In this paper, four different mechanical model of a balancing task is considered as a special type of tracking problems. First, it is shown that all the four systems are governed by the same second-order system (with different meaning of the system parameters). Then, the stabilizability conditions are derived in closed form in case of digital PIDA feedback.

2. Modeling

Four different mechanical models shown in Figure 1 are investigated. Panel I) illustrates the control of a block of mass in the vertical plane. The control input is a vertical force and the control goal is the stabilization of the vertical position of the mass about $\phi = 0$. In panels II)–IV) three different mechanical systems are shown, each incorporating the balancing of a homogeneous, prismatic beam. In each case the control goal is the stabilization of the beam about its upper or lower equilibrium points. In panel II), the control input is a torque, which is applied directly on the beam e.g., by a rotating inertial disc. In contrast, in panels III) and IV) the control input acting on the beam is a force, which is produced by a sliding cart (panel III) and by a rolling wheel (panel IV). In the latter case, the actual control effort is a torque acting on the wheel.

Without going into details on the derivation of the equations of motion, the governing equations of the mechanical systems in Figure 1 are listed below.

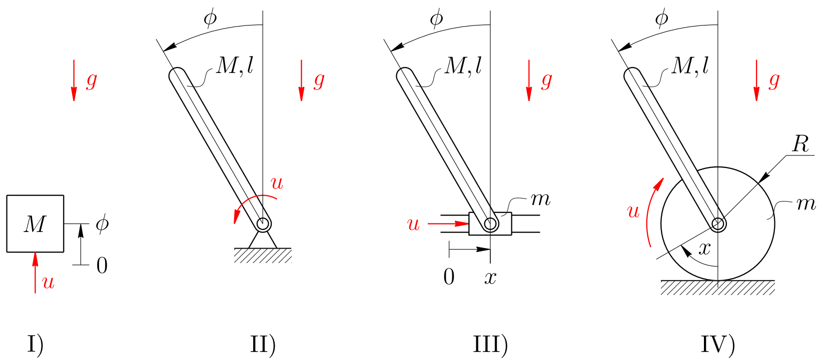

Fig. 1. The analyzed mechanical models

I)

$$\ddot{\phi} = \frac{u}{M} - g \tag{1}$$

II)

$$\ddot{\phi} - \frac{3g}{2l}\sin(\phi) = \frac{3u}{Ml^2} \tag{2}$$

III)

$$2l\ddot{\phi} - 3g\sin(\phi) - 3\cos(\phi)\,\ddot{x} = 0 \tag{3}$$

$$-lM\ddot{\phi}\cos(\phi) + lM\dot{\phi}^2\sin(\phi) + 2(M+m)\ddot{x} = 2u \tag{4}$$

IV)

$$2l\ddot{\phi} - 3g\sin(\phi) - 3R\cos(\phi)\,\ddot{x} = 0 \tag{5}$$

$$-lRM\ddot{\phi}\cos(\phi) + lRM\dot{\phi}^2\sin(\phi) + R^2(2M+3m)\,\ddot{x} = 2u \tag{6}$$

Here M is the mass of the block (I) and the pendulum (II-IV), l is the length if the pendulum, m is the mass of the sliding cart (III) and the wheel (IV) and R is the radius of the wheel, which is assumed to be homogeneous. The general coordinate to be controlled is ϕ while x is a cyclic coordinate representing the position of the suspension point of the pendulum (III, IV). The control force/torque is denoted by u.

Note that the mechanical systems all have ideal constraints, thus the above equations of motion can be derived using Lagrange's equations of the second kind. Also note that, for model III, $\ddot{x}$ can be expressed from (3) (or from (4)) and by substituting it to (4) (or to (3)) one can obtain a differential equation which is solely dependent on ϕ and its derivatives:

$$\left(1 - \frac{3M}{4(M+m)}\cos^2(\phi)\right)\ddot{\phi} + \frac{3M}{4(M+m)}\sin(\phi)\cos(\phi)\,\dot{\phi}^2 - \frac{3g}{2l}\sin(\phi) = \frac{3\cos(\phi)\,u}{2l(M+m)}. \tag{7}$$

In a similar way, for model IV, the equation of motion can be derived using (5)–(6) in the form

$$\left(1 - \frac{3M}{4M+6m}\cos^2(\phi)\right)\ddot{\phi} + \frac{3M}{4M+6m}\sin(\phi)\cos(\phi)\,\dot{\phi}^2 - \frac{3g}{2l}\sin(\phi) = \frac{3\cos(\phi)\,u}{lR(2M+3m)}. \tag{8}$$

After the linearization of (2), (7) and (8) about the equilibrium points $\phi_e = 0$ and $\phi_e = \pi$, the governing equation of (1) and the linearized equations of (2), (7) and (8) can be summarized in the form

$$\ddot{\psi}(t) + s\psi(t) = qu(t) + r, \tag{9}$$

where $\psi = \phi - \phi_e$ is a local coordinate about the equilibrium point,

$$r = \begin{cases} 0 & \text{for models II), III) and IV)} \\ -g & \text{for model I)} \end{cases} \tag{10}$$

coefficients s and q are summarized in Table 1, and $s_0 = 3g/(2l)$.

The control input is given by a PIDA feedback rule of the form

$$u(t) = -\mathrm{sgn}(q)\left(I\Psi_{j-1} + P\psi\left(t_{j-1}\right) + D\dot{\psi}\left(t_{j-1}\right) + A\ddot{\psi}\left(t_{j-1}\right) + qr\right), \quad t \in \left[t_j, t_{j+1}\right), \tag{11}$$

where I, P, D and A are the integral, the proportional, the derivative and the acceleration feedback gains, respectively, Ψ_j denotes the numerical integral of $\psi(t)$ calculated at $t = t_{j+1}$ by the rectangle rule

$$\Psi_j = \Psi_{j-1} + \Delta t\,\psi\left(t_j\right). \tag{12}$$

Note that in addition to the feedback term, a feedforward term qr is also applied in the control law (11) in order to eliminate the explicit terms related to the external forces.

	ϕ_e	I)	II)	III)	IV)
s	0	0	$-s_0$	$-s_0 \frac{4M+4m}{M+4m}$	$-s_0 \frac{4M+6m}{M+6m}$
	π		s_0	$s_0 \frac{4M+4m}{M+4m}$	$s_0 \frac{4M+6m}{M+6m}$
q	0	$\frac{1}{M}$	$\frac{3}{Ml^2}$	$\frac{6}{l(M+4m)}$	$\frac{6}{lR(M+6m)}$
	π			$-\frac{6}{l(M+4m)}$	$-\frac{6}{lR(M+6m)}$

Table 1. Parameters s and q within (9) for different mechanical systems, about different ϕ_e equilibrium states

Equation (11) assumes that the position, velocity and acceleration of the local coordinate ψ are measured accurately with sampling frequency f_s. Therefore, measurements are carried out in each $t_j = j\Delta t$ time instant, where $\Delta t = 1/f_s$ is the sampling period and $j \in \mathbb{N}$. Equation (11) further assumes that the processing of measured data (e.g. filtering) and the time necessary for the computation of the control force together are less than a sampling period. This implies that the applied control force in time period $t \in [t_j, t_{j+1}]$ is based on the data measured at time instant t_{j-1}. Since the control input can be updated only upon the arrival of the next measurement, its value is held constant between two consecutive samples, which introduce a zero-order hold in the feedback loop. Together, the zero-order hold and the delay between measurement and actuation result in a time-varying input delay which increases linearly from Δt to $2\Delta t$ within each sampling period $t \in [t_j, t_{j+1})$. Further details on the above described sampling concept and actuation scheme can be found in the article by Stepan [10].

After rescaling the time as $\tilde{t} = |s|^{1/2} t$, introducing dimensionless variables $\Delta\tilde{t} = |s|^{1/2}\Delta t$, $\tilde{\Psi}_j = |s|^{1/2}\Psi_j$, $k_I = Iq|s|^{-3/2}$, $k_P = Pq|s|^{-1}$, $k_D = Dq|s|^{-1/2}$, $k_A = Aq$ and dropping the tildes immediately, (9), (11) and (12) result in the hybrid system

$$\ddot{\psi}(t) + \operatorname{sgn}(s)\psi(t) = -k_I\Psi_{j-2} - (k_P + k_I\Delta t)\,\psi\big(t_{j-1}\big) - k_D\dot{\psi}\big(t_{j-1}\big) - k_A\ddot{\psi}\big(t_{j-1}\big), \quad t \in \big[t_j, t_{j+1}\big), \tag{13}$$

$$\Psi_{j-1} = \Psi_{j-2} + \Delta t\,\psi\big(t_{j-1}\big). \tag{14}$$

3. Stability analysis

The stability analysis of the hybrid system (13)–(14) provides information on the permissible control gains which assure the stability of the closed-loop system about the investigated equilibrium points. Due to the sampling effect, the analyzed hybrid system is equivalent to a time-periodic ordinary differential equation with principal period Δt, hence its stability is determined by its monodromy matrix. In what follows, first, the derivation of the monodromy matrix is given, then, based on the eigenvalues of the monodromy matrix (which are also called characteristic multipliers), necessary and sufficient conditions are derived for the stability and stabilizability of the hybrid system. For the construction of the monodromy matrix one has to determine a mapping for the state variables of (13)–(14) between two consecutive samples at sampling instants t_j and t_{j+1}.

By denoting the right-hand side of (13) as

$$U(t) = -k_I\Psi_{j-2} - (k_P + k_I\Delta t)\,\psi\big(t_{j-1}\big) - k_D\dot{\psi}\big(t_{j-1}\big) - k_A\ddot{\psi}\big(t_{j-1}\big), \quad t \in \big[t_j, t_{j+1}\big), \tag{15}$$

the solution of (13) with initial conditions

$$\psi\big(t_j\big) = \psi_j, \quad \dot{\psi}\big(t_j\big) = \dot{\psi}_j, \quad U\big(t_j\big) = U_j \tag{16}$$

can be uniquely determined in $t \in \big[t_j, t_{j+1}\big)$. In the following this solution is denoted by $\chi_j\big(t, \psi_j, \dot{\psi}_j, U_j\big)$. Note that solution χ_j depends always linearly on ψ_j, $\dot{\psi}_j$ and U_j. Since (15) contains no impact-like term, the solution and its first derivative are both continuous, that is

$$\psi_{j+1} = \chi_{j+1}\left(t_{j+1}, \psi_{j+1}, \dot{\psi}_{j+1}, U_{j+1}\right) = \chi_j\left(t_{j+1}, \psi_j, \dot{\psi}_j, U_j\right), \tag{17}$$

$$\dot{\psi}_{j+1} = \dot{\chi}_{j+1}\left(t_{j+1}, \psi_{j+1}, \dot{\psi}_{j+1}, U_{j+1}\right) = \dot{\chi}_j\left(t_{j+1}, \psi_j, \dot{\psi}_j, U_j\right). \tag{18}$$

Consequently, after specifying initial conditions Ψ_{-1}, ψ_0, $\dot{\psi}_0$ and U_0 one can compose a "global" solution of (13)–(14) as

$$\chi\left(t, \Psi_{-1}, \psi_0, \dot{\psi}_0, U_0\right) = \left\{\left(\chi_j\left(t, \psi_j, \dot{\psi}_j, U_j\right), \Psi_{j-1}\right), t \in \left[t_j, t_{j+1}\right), j \in \mathbb{N}\right\}, \tag{19}$$

by solving (13)–(14) from interval to interval.

Note, however, that there is an important modeling issue related to the feedback of discrete values of the acceleration, namely, the second derivative of the solution χ_j is subjected to discontinuities at the sampling instants due to the discontinuities in the control force, which occur at each instant when the control force is updated. The discontinuities make the feedback of the acceleration ambiguous, since measurements can be made right before (at $t = t_j^-$) or right after (at $t = t_j^+$) the update in the control force. In the case when the measurement of the acceleration is made right before the update in the control force, one measures

$$\ddot{\psi}^-_{j+1} = \ddot{\chi}_j\left(t^-_{j+1}, \psi_j, \dot{\psi}_j, U^-_j\right), \tag{20}$$

where

$$U^-_j = -k_I\Psi_{j-2} - (k_P + k_I\Delta t)\,\psi_{j-1} - k_D\dot{\psi}_{j-1} - k_A\ddot{\psi}^-_{j-1}. \tag{21}$$

On the other hand, when the acceleration is measured right after the update in the control force, one has

$$\ddot{\psi}^+_{j+1} = -\mathrm{sgn}(s)\psi_{j+1} + U^+_{j+1}, \tag{22}$$

with

$$U^+_{j+1} = -k_I\Psi_{j-1} - (k_P + k_I\Delta t)\,\psi_j - k_D\dot{\psi}_j - k_A\ddot{\psi}^+_j. \tag{23}$$

In both cases a mapping can be constructed of the form

$$\mathbf{Y}_{j+1} = \mathbf{\Phi}\mathbf{Y}_j, \tag{24}$$

which relates the generalized state variables at time instants t_j and t_{j+1}.

In the case when the measurement is made right before the update in the control force, this mapping is composed from equation (14), (17)–(18) and (20)–(21). The generalized state variables are

$$\mathbf{Y}^-_j = \begin{bmatrix}\Psi_{j-1} & \psi_j & \dot{\psi}_j & \ddot{\psi}^-_j & U^-_j\end{bmatrix}^\mathsf{T} \tag{25}$$

and the monodromy matrix is

$$\mathbf{\Phi}^- = \begin{bmatrix} 1 & \Delta t & 0 & 0 & 0 \\ 0 & a & b & 0 & c \\ 0 & -\mathrm{sgn}(s)b & a & 0 & b \\ 0 & -\mathrm{sgn}(s)a & -\mathrm{sgn}(s)b & 0 & a \\ -k_I & -(\Delta t k_I + k_P) & -k_D & -k_A & 0 \end{bmatrix}, \tag{26}$$

where parameters a, b and c are given in Table 2.

In the case when the measurement is made right after the update in the control force, the mapping is constructed from (14), (17)–(18) and (22). The generalized state variables are

$$\mathbf{Y}^+_j = \begin{bmatrix}\Psi_{j-1} & \psi_j & \dot{\psi}_j & \ddot{\psi}^+_j\end{bmatrix}^\mathsf{T} \tag{27}$$

sgn(s)	a	b	c
-1	$\cosh(\Delta t)$	$\sinh(\Delta t)$	$a-1$
0	1	Δt	$\Delta t^2/2$
1	$\cos(\Delta t)$	$\sin(\Delta t)$	$1-a$

Table 2. Parameters within monodromy matrices (26) and (28)

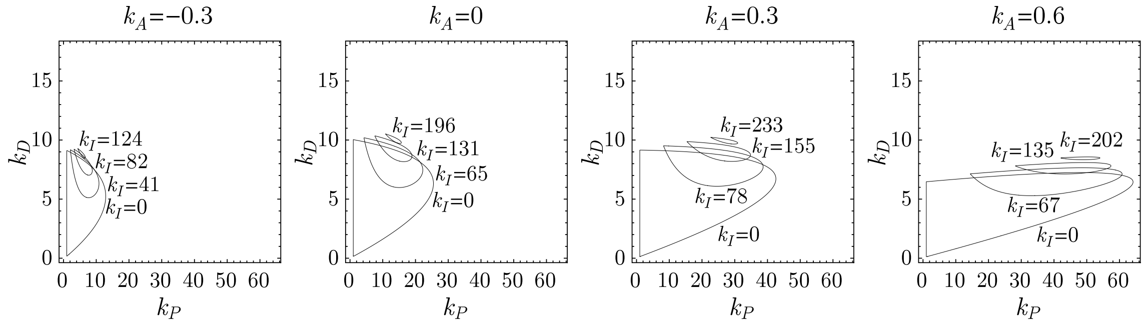

Fig. 2. Stability diagrams for the case $s < 0$ and $\Delta t = 0.1$. The boundary of stability is depicted for different acceleration and integral control gains in the case when acceleration is measured right before the update in the control force.

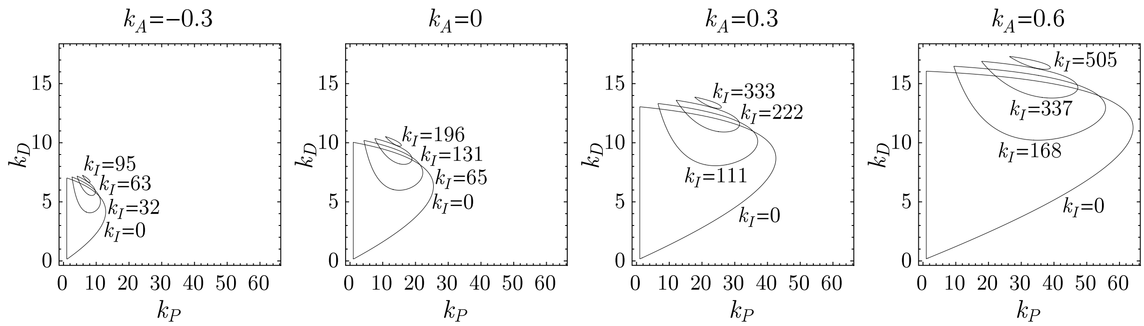

Fig. 3. Stability diagrams for the case $s < 0$ and $\Delta t = 0.1$. The boundary of stability is depicted for different acceleration and integral control gains in the case when acceleration is measured right after the update in the control force.

and the monodromy matrix is

$$\mathbf{\Phi}^+ = \begin{bmatrix} 1 & \Delta t & 0 & 0 \\ 0 & 1 & b & c \\ 0 & 0 & a & b \\ -k_I & -(\Delta t k_I + k_P) - \mathrm{sgn}(s) & -k_D - \mathrm{sgn}(s) b & -k_A - \mathrm{sgn}(s) c \end{bmatrix}. \tag{28}$$

Parameters a, b and c are again given in Table 2.

Note, that in reality, the updated control force does not prevail immediately, consequently the measurement of acceleration right before the update in the control force described by monodromy matrix (26) is more realistic.

Mapping (24) is stable if and only if the absolute value of all eigenvalues μ_i (also called as characteristic multipliers) of monodromy matrix $\mathbf{\Phi}$ are less than one, that is $|\mu_i| < 1$, $\forall i$. Stability domains can be determined by checking this condition numerically over a grid in the space of control parameters. In Figures 2 and 3, the stability boundaries are visualized for monodromy matrices (26) and (28), respectively, for different acceleration feedback gains, fixed sampling period and $s < 0$. The stable domains are those within the closed curves.

Other than numerical results, a more qualitative analysis can be carried out and the stability boundaries can be derived in a closed form by following the same steps detailed by Stepan and Enikov [14,10]. In particular by employing Möbius transformation the stability criterion $|\mu_i| < 1$, $\forall i$ can be transformed to the Liénard–Chipart stability criterion. Below this derivation is omitted for the sake of brevity. The results can be summarized as follows. With the assumption that measurements are made right before the update in the control force, the hybrid system (13)–(14) is stable if and only if all the following inequalities hold:

$$a_5 > 0, \tag{29}$$

$$a_3 > 0, \tag{30}$$

$$a_1 > 0, \tag{31}$$

$$a_1 a_2 - a_0 a_3 > 0, \tag{32}$$

$$-a_1^2 a_4^2 + a_1\Big(a_2 a_3 a_4 - a_2^2 a_5 + 2a_0 a_4 a_5\Big) - a_0\Big(a_3^2 a_4 - a_2 a_3 a_5 + a_0 a_5^2\Big) > 0, \tag{33}$$

where coefficients a_i, $i = 0, \dots, 5$ are

$$a_0 = \Delta t\, k_I \left(-ac + b^2 + c\right), \tag{34}$$

$$\begin{aligned} a_1 = {} & 2a^3 k_A + 2a^2(k_A(2-\mathrm{sgn}(s)c) - 1) - a\left(4 + 2bk_D + 2ck_P - c\Delta t k_I - 2k_A\Big(\mathrm{sgn}(s)b^2 - \mathrm{sgn}(s)c + 1\Big)\right) \\ & - b^2(\Delta t k_I - 2(ck_A - \mathrm{sgn}(s)k_A + k_P + \mathrm{sgn}(s))) - 2bk_D(\mathrm{sgn}(s)c - 1) + c\Delta t k_I + 2ck_P + 2, \end{aligned} \tag{35}$$

$$\begin{aligned} a_2 = {} & 2\mathrm{sgn}(s)bck_D + 4 - c\Delta t k_I - 4a^3 k_A - 4a^2 k_A(\mathrm{sgn}(s)c - 1) - a\Big(4\mathrm{sgn}(s)b^2 k_A - 2bk_D - c\,(\Delta t k_I + 2\mathrm{sgn}(s)k_A + 2k_P) + 4\Big) \\ & - b^2(2\mathrm{sgn}(s)k_A(2\mathrm{sgn}(s)c - 1) + 2k_P + \Delta t k_I)\,, \end{aligned} \tag{36}$$

$$\begin{aligned} a_3 = {} & 6a^3 k_A + a^2(6\mathrm{sgn}(s)ck_A - 2) - a\left(2k_A - 6\mathrm{sgn}(s)b^2 k_A + c\Delta t k_I\right) \\ & + b^2(6ck_A + \Delta t k_I - 2\mathrm{sgn}(s)) - 2bk_D - c\Delta t k_I - 2ck_P + 6, \end{aligned} \tag{37}$$

$$\begin{aligned} a_4 = {} & -8a^3 k_A - 8a^2 k_A(\mathrm{sgn}(s)c + 1) - a\Big(4\mathrm{sgn}(s)k_A\left(2b^2 + c\right) + 4bk_D + c\Delta t k_I + 4ck_P - 8\Big) \\ & - b^2\left(4\mathrm{sgn}(s)k_A\left(2\mathrm{sgn}(s)c + 1\right) - \Delta t k_I - 4k_P\right) - 4\mathrm{sgn}(s)bck_D + c\Delta t k_I + 8, \end{aligned} \tag{38}$$

$$\begin{aligned} a_5 = {} & 2a^3 k_A + 2a^2\left(k_A\left(\mathrm{sgn}(s)c + 2\right) + 1\right) + a\left(2k_A\left(\mathrm{sgn}(s)\left(b^2 + c\right) + 1\right) + 2bk_D + c\Delta t k_I + 2ck_P + 4\right) \\ & - b^2\left(\Delta t k_I - 2\left(\mathrm{sgn}(s)\left(\mathrm{sgn}(s)ck_A + k_A + 1\right) - kp\right)\right) + 2b\left(\mathrm{sgn}(s)ck_D + k_D\right) + c\Delta t k_I + 2ck_P + 2. \end{aligned} \tag{39}$$

In addition to the above analytical results, authors make the following conjecture based on numerical stability tests. With the assumption that measurements are made right before the update in the control force, the hybrid system (13)–(14), with $k_I \neq 0$ and $s \leq 0$ is stabilizable if and only if conditions

$$0 \leq k_I \leq \frac{(1 - k_A)\,(3 + k_A - 2a)^2}{8c\Delta t} \quad \text{and} \quad 2a - 3 \leq k_A \leq 1 \tag{40}$$

hold. For the case $s < 0$, the limit of stabilizability is depicted in Figure 4. The stabilizable domains are those below the curves. It can be seen that the large sampling periods are associated with zero integral gain, that is, the inclusion of the integral term in the feedback loop does not contribute to the stabilizability.

4. Conclusions

This paper dealt with the stability analysis of four simple mechanical systems subjected to digital feedback control. The stability and stabilizability conditions of these mechanical systems were studied by means of the analysis of their governing equations linearized about their equilibrium points. In a unified framework of the mechanical models, an PIDA feedback rule was analyzed which took into account the sampling effect of the feedback loop and the zero order hold of the controller. For some specific cases closed form formulas were derived for the domains of stability and stabilizability. Overall, it can be concluded that the integral gain can only deteriorate the performance of the controller since it always decreases the domain of stability.

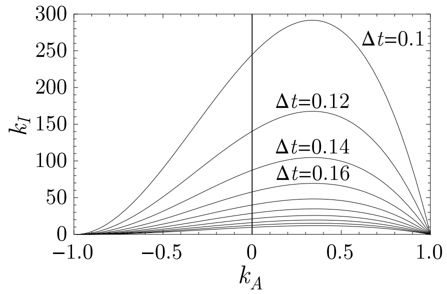

Fig. 4. Stabilizability limits for $k_I \neq 0$, $s < 0$ and for the case when acceleration is sampled right before actuation.

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