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Stick balancing with feedback delay, sensory dead zone, acceleration and jerk limitation

Tamas Insperger*[a], John Milton[b]

[a]Department of Applied Mechanics, Budapest University of Technology and Economics and MTA-BME Lendüulet Human Balancing Research Group, 1111 Budapest, Hungary

[b]W. M. Keck Science Department, The Claremont Colleges, Claremont, CA 91711, USA

Abstract

A simplified model of stick balancing on the fingertip subjected to predictor feedback is investigated, which accounts for three important modeling issues: (1) feedback delay; (2) the sensory dead zone; and (3) limitation of the control force corresponding to the maximum acceleration and the maximum jerk of human hand movement. Eight different cases (± sensory dead zone, ± acceleration limitation, ± jerk limitation) are compared by estimating the maximum balance time out of five time-domain simulations with different initial conditions. It is shown that the region of linear stability in the plane of control parameters is reduced by the presence of dead zone, not affected by limitations on hand acceleration, but is increased by limitations on the jerk.

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Keywords: feedback delay, stability, bounded motions, control force saturation, jerk.

1. Introduction

The most difficult motor tasks to control are those which involve the stabilization of an unstable position. Consequently studies of stick or pole balancing at the fingertip are important for investigating the neural control of voluntary movements and the development of skill with practice [1,2,3,4,5,6,7,8]. Two different perspectives have arisen concerning the nature of the underlying neural control strategy. First, studies which focus on stability of the upright position emphasize the importance of model predictive, or forward, control mechanisms [2]. For a given feedback delay, the more robust mechanisms are those that can balance the shortest stick [9]. Second, studies that focus on the dynamics exhibited by the balanced stick emphasize the presence of a sensory dead zone, namely the existence of a range of sensory input for which no corrective actions are taken [10,11]. From a mathematical point of view a dead zone is a strong, small-scale nonlinearity which has no effect on the large-scale stabilization of the linear dynamical system, but has major effects on the generation of complex dynamics such as limit cycles and chaos [12,13].

* Corresponding author.
E-mail address: insperger@mm.bme.hu

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Recently we showed that expert stick balancers are able to balance a 30cm stick on their fingertip for 240s and that the time delay, τ, was 230 ms [14]. These observations argue in favor of a predictor-type feedback control mechanism for stick balancing in 3D. Here we explore this model further by examining the effects of limitations on the maximum acceleration and maximum jerk of the fingertip. These limitations represent an additional constraint on this balancing task, namely, a saturation of the control force. Although the role of the jerk has received no previous attention with respect to stick balancing, its role in other visually guided hand movements is well recognized [15].

In this paper we demonstrate that traditional time-delayed PD controller cannot stabilize sticks of length 30cm when τ = 230ms for 240s. Numerical evidence in support of a pendulum-cart model for stick balancing using a predictor feedback control mechanism is provided. For this model, the inertia of arm mechanism is transformed into the mass of the cart. Diagrams representing the balance time for different control parameter combinations are presenting for different stick length. It is shown that as a result of the interaction of feedback delay, sensory dead zone and fingertip movement constraints, a stick shorter than about 30cm cannot be balanced longer than 240s as observed experimentally.

2. Mechanical model with arm mechanism

Balancing tasks are often modeled by the pendulum-cart model, which accounts for the fact that the hand is moving back and forth in the anteriorposterior direction. The equivalence between the human arm mechanism and the pendulum-cart model is to be established by setting the mass m_0 of the cart according to the inertia of the arm segments. During stick balancing on the fingertip, the movements of the arm of an experienced stick balancer are confined to the elbow and shoulder while the wrist and finger are held rigid [3]. Assuming that the hand moves horizontally, the human arm can be modeled as a slider crank mechanism shown in Fig. 1. The parameters of the mechanism are set according to the average human arm segment as listed in Table 1 [16].

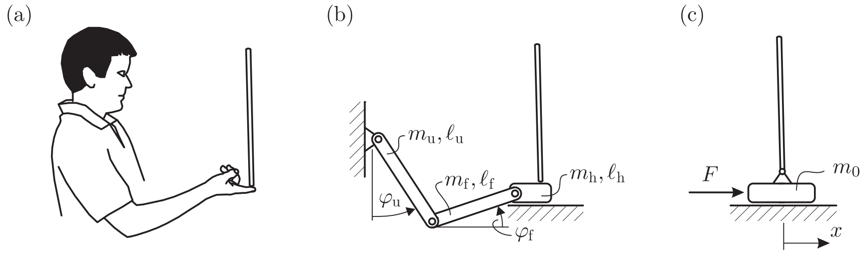

Fig. 1. Model of human arm mechanism.

Table 1. Arm segment parameters taken from [16].

segment	mass	length (t)
upper arm	m_u = 1.775kg	ℓ_u = 0.2874m
forearm	m_f = 1.015kg	ℓ_f = 0.2666m
hand	m_h = 1.015kg	ℓ_h = 0.0821m

The equivalence of the models is based on the equivalence of their kinetic energy. The kinetic energy of the arm mechanism can be given as

$$E_{\text{kin,arm}} = \frac{1}{2}m_u v_u^2 + \frac{1}{2}J_u\dot{\varphi}_u^2 + \frac{1}{2}m_f v_f^2 + \frac{1}{2}J_f\dot{\varphi}_f^2 + \frac{1}{2}m_h v_h^2, \tag{1}$$

where v_u, v_f and v_h are the velocities of the center of gravity of the upper arm, forearm and hand, respectively, and J_u and J_f are the moment of inertia with respect to the normal line via the center of gravity of the upper arm and the forearm. Assuming homogeneous arm segments, we have $J_u = \frac{1}{12}m_u\ell_u^2$ and $J_f = \frac{1}{12}m_f\ell_f^2$. Since the mass of the stick

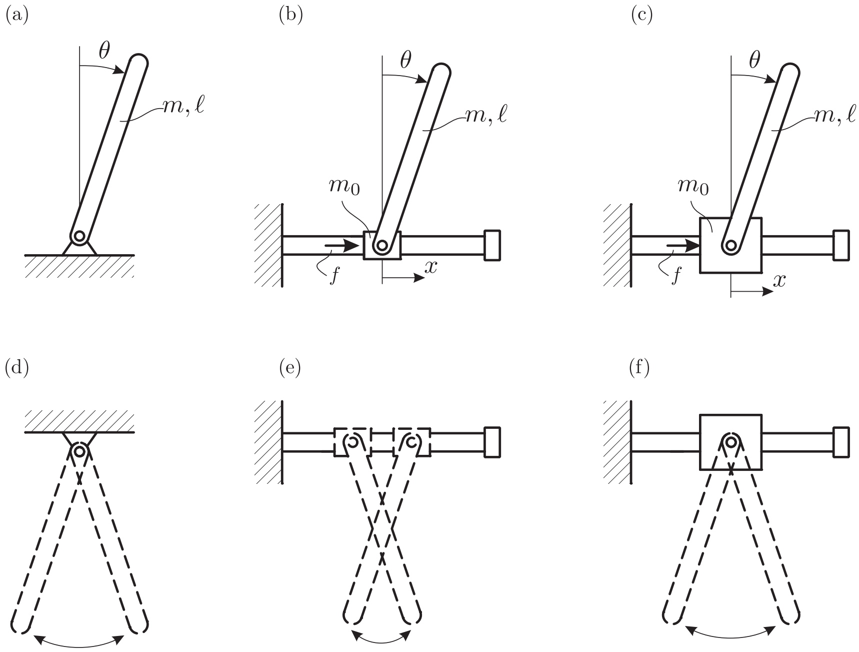

Fig. 2. Mechanical model of (a) pinned pendulum; (b) pendulum-cart model with negligible cart mass ($m_0 \ll m$); (c) pendulum-cart model with $m_0 \gg m$. Panels (d), (e), (f) shows the oscillations when the pendulums are hung downward.

($m = 0.005 \sim 0.022$kg) is negligible compared to the mass of the arm-hand mechanism (cart), the kinetic energy of the pendulum cart model is

$$E_{\mathrm{kin,cart}} = \frac{1}{2} m_0 \dot{x}^2. \tag{2}$$

Using the equations $E_{\mathrm{kin,arm}} = E_{\mathrm{kin,cart}}$, $v_{\mathrm{h}} = \dot{x}$ and the relations between the velocities v_{u}, v_{f} and v_{h} and the angular velocities $\dot{\varphi}_{\mathrm{u}}$ and $\dot{\varphi}_{\mathrm{f}}$, one can calculate the equivalent mass m_0 of the cart. Assuming $\varphi_{\mathrm{u}} \approx 20$deg and $\varphi_{\mathrm{u}} \approx 10$deg for the arm segment positions during stick balancing, the mass of the cart is obtained to be $m_0 = 1.2$kg.

3. Stabilizability and critical lengths for delayed PD feedback

The differences between models with different m_0 and their connection to the pinned inverted pendulum model is illustrated in Fig. 2. The governing equations for the pinned inverted pendulum shown in Fig. 2a is

$$\ddot{\theta}(t) - \frac{3g}{2\ell}\theta(t) = -\frac{3}{m\ell^2} T, \tag{3}$$

where θ denotes the angular deviation of the pendulum from vertical, ℓ is the length of the pendulum and T is the control torque. When the pinned pendulum is hung downward its oscillation period is equal to

$$T_{\mathrm{d}} = 2\pi \sqrt{\frac{2\ell}{3g}}. \tag{4}$$

When the position of the cart is not controlled, then the governing equations for the pendulum-cart model shown in Fig. 2b and Fig. 2c is

$$\ddot{\theta}(t) - \frac{6g}{c\ell}\theta(t) = -\frac{6}{(m + m_0)c\ell} F, \tag{5}$$

where $c = 4 - 3m/(m + m_0)$ is a constant and F is the control force. When the mass of the cart is negligible ($m_0 \ll m$, see Fig. 2b), then the constant $c = 1$. In this case, the oscillation period in the downward position is

$$T_e = 2\pi\sqrt{\frac{\ell}{6g}} = \frac{1}{2}T_d, \tag{6}$$

which is the half of the oscillation period of the pinned pendulum. When the mass of the cart is significantly larger than the mass of the pendulum ($m_0 \gg m$, see Fig. 2c), then $c = 4$. In this case, the oscillation period in the downward position is

$$T_f = 2\pi\sqrt{\frac{2\ell}{3g}} = T_d, \tag{7}$$

which is just equal to the oscillation period of the pinned pendulum.

It is known that structures cannot be stabilized about their unstable equilibria via delayed PD feedback if the delay is larger than the critical value

$$\tau_{\text{crit,PD}} = \frac{T_p}{\pi\sqrt{2}}, \tag{8}$$

where T_p is the period of the small oscillations of the structure hung at its downward position [17]. This implies that there is a significant difference between the pinned inverted pendulum (or the pendulum-cart model with $m_0 \gg m$) and the pendulum-cart model with $m_0 \ll m$. For the pinned inverted pendulum (Fig. 2a) or for the pendulum-cart model with $m_0 \gg m$ (Fig. 2c), the critical delay is

$$\tau_{\text{crit,PD,a,c}} = \frac{T_d}{\pi\sqrt{2}} = \sqrt{\frac{4\ell}{3g}}. \tag{9}$$

Alternatively, stabilizability condition can be defined as a critical length for a fixed feedback delay τ as

$$\ell_{\text{crit,PD,a,c}} = \frac{3}{4}g\tau^2. \tag{10}$$

For the pendulum-cart model with $m_0 \ll m$ (Fig. 2b), the critical length is

$$\ell_{\text{crit,PD,b}} = 3g\tau^2. \tag{11}$$

Thus the critical length for the pendulum-cart model with $m_0 \gg m$ is quarter of that with $m_0 \ll m$. For instance, for stick balancing at the fingertip, where the feedback delay is estimated to be $\tau = 230$ms [14], the critical length for the case $m_0 \ll m$ is $\ell_{\text{crit,PD,b}} = 156$cm, while for the case $m_0 \gg m$ is $\ell_{\text{crit,PD,c}} = 39$cm only.

One should note that the above models assume a perfect implementation of the control loop free of any uncertainties or noise. In real stick balancing, the critical length is certainly larger than the ones above due to the uncertainties in the sensory inputs, the imperfect implementation of the control law and the noise in the actuation force. Experimental stick balancing trials showed that skilled subjects are still able to balance sticks shorter than 30cm for minutes [14]. The above observations imply that the control concept of the nervous system during stick balancing is not a PD feedback, but something more efficient and sophisticated control algorithm, e.g., PDA feedback [9,18], intermittent feedback [4,7,8], predictor feedback [2,9,14]. In the next section, we assume predictor feedback subjected to movement constraints of the cart (fingertip).

4. Predictor feedback model with movement constraints on the displacement of the cart

If the displacement x of the cart is to be controlled, then the system cannot be reduced to a second-order scalar equation as in (5). In this case the first-order representation of the pendulum-cart system reads

$$\dot{z}(t) = Az(t) + BF(t), \tag{12}$$

where

$$z(t) = \begin{pmatrix} \theta(t) \\ x(t) \\ \dot{\theta}(t) \\ \dot{x}(t) \end{pmatrix}, \quad A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \multicolumn{2}{c}{-M^{-1}K} & 0 & 0 \\ & & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ M^{-1}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{pmatrix} \tag{13}$$

with

$$M = \begin{pmatrix} \frac{1}{3}m\ell^2 & \frac{1}{2}m\ell \\ \frac{1}{2}m\ell & m+m_0 \end{pmatrix}, \quad K = \begin{pmatrix} -\frac{1}{2}mg\ell & 0 \\ 0 & 0 \end{pmatrix} \tag{14}$$

being the mass matrix and the stiffness matrix, respectively. Predictor feedback [19] requires the prediction

$$z_{\rm pred}(t) = \mathrm{e}^{\tilde{A}\tilde{\tau}} z(t-\tau) + \int_{t-\tilde{\tau}}^{t} \mathrm{e}^{\tilde{A}(t-s)} \tilde{B} f_{\rm PF}(s)\mathrm{d}s, \tag{15}$$

where $\tilde{A}$, $\tilde{B}$ and $\tilde{\tau}$ are the parameters of the internal model of the neural system. This prediction is then employed for a PD feedback, thus

$$F_{\rm PF}(t) = K z_{\rm pred}(t), \tag{16}$$

with

$$K = \begin{pmatrix} k_{{\rm p},\theta} & k_{{\rm p},x} & k_{{\rm d},\theta} & k_{{\rm d},x} \end{pmatrix}. \tag{17}$$

If the internal model perfectly matches the real system, i.e., $\tilde{A} = A$, $\tilde{B} = B$ and $\tilde{\tau} = \tau$, then the prediction is perfect, $z_{\rm pred}(t) = z(t)$, and the feedback delay is eliminated from the control loop. For stick balancing, perfect matching of the internal model means that the masses m and m_0, the length ℓ of the stick and the feedback delay τ are known. In the subsequent analysis, we assume that these parameters are known as a result of a long enough learning process (practice).

Linear stability analysis of the system can be performed by standard techniques, such as the semidiscretization method [20]. However, constraints on the control action and imperfections of the sensory perception present strong nonlinearities in the governing equations, and linear stability predictions are not valid any more. In this cases, the response of the system can be analyzed by numerical simulations. Here the following constraints were involved into the model.

- Sensory dead zone for the angular displacement θ. We assume that the angular position perceived by the neural system is

$$\theta_{\rm perceived}(t-\tau) = \begin{cases} 0 & \text{if } |\theta_{\rm a}(t-\tau)| < \Pi \\ \theta_{\rm a}(t-\tau) & \text{if } |\theta_{\rm a}(t-\tau)| \geq \Pi \;. \end{cases} \tag{18}$$

  where $\theta_{\rm a}$ is the stick's actual angle and Π is the functional sensory threshold. Experimental estimations give $\Pi = 0.8$deg for expert stick balancers [14].
- Constraint on the maximum fingertip acceleration. We assume that the maximum control force is limited by $m_0 a_{\rm max}$ where $a_{\rm max} \approx 50{\rm m/s}^2$ is the maximum acceleration of the fingertip.
- Constraint on the maximum fingertip jerk. We assume that the rate of change of the control force is limited by $m_0 j_{\rm max}$, where experimental observations suggest that $j_{\rm max} \approx 600{\rm m/s}^3$ [21].

Stabilizability conditions were investigated by systematic numerical simulations for a range of control parameters. For each parameter combinations, numerical simulation was performed for five different initial conditions while $|x|$ and $|\theta|$ was monitored. We compared the effects of these constraints on stick balancing by estimating the maximum balance time (BT), namely, the maximum balance time from five time-domain simulations with different initial conditions. A stick was considered to be balanced if $|\theta| \leq \theta_{\rm lim}$ and $x \leq x_{\rm lim}$. During expert stick balancing θ never exceeds 20deg. There we took $\theta_{\rm lim} = 20$deg. When x exceeds $x_{\rm lim}$, the stick is out of the reach of the subject's arm. Therefore we took $x_{\rm lim}$ to be equal to the half-arm length, i.e. 0.335m [21]. The simulations were terminated if at least one out of the five trials lasted for 240s. For a given parameter combination, the recorded balance time (BT) was the duration of the longest simulation.

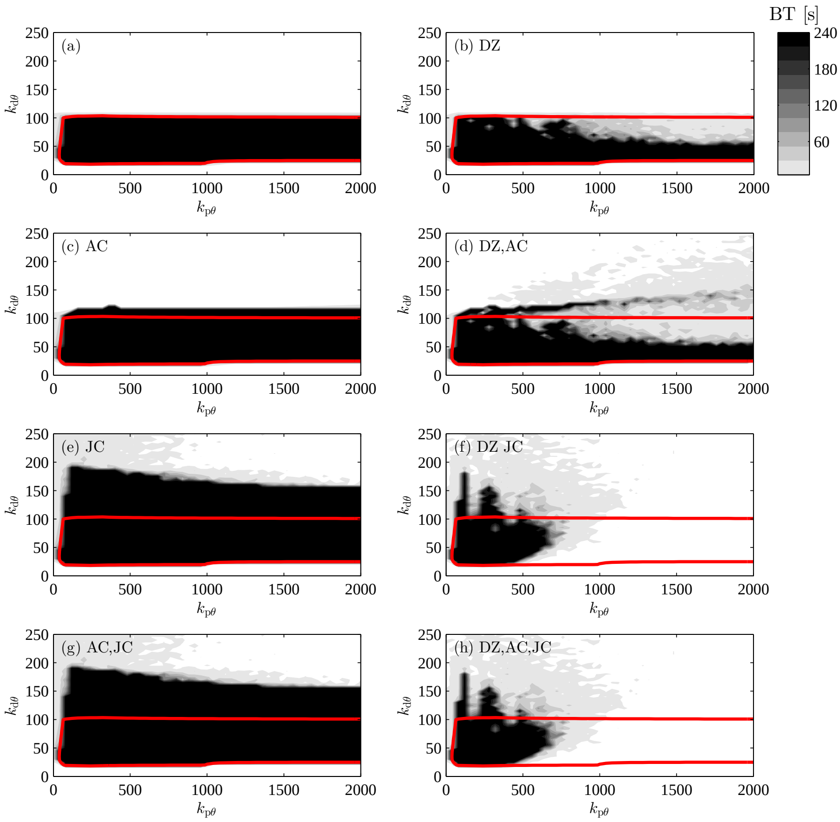

Fig. 3. Effect of sensory deadzone (DZ) and limitations on the maximum acceleration (AC) and jerk (JC) on balance time (BT). The maximum BT for five trials with different initial conditions is represented by the gray scale shown on the right. The linear stability boundary for the pendulum-cart model with predictor feedback in the absence of constraints is given by the red line. (a), (c), (e), (g) are without dead zone, (a), (b), (e), (f) are without acceleration limitation and (a), (b), (c), (d) are without jerk limitation. For all simulations we took $k_{p,x} = 20$, $k_{d,x} = 40$.

The results are summarized in Fig. 3. Cases with and without dead zone, with and without acceleration limitation and with and without jerk limitation are considered, which give eight different cases. The stability boundaries for the linear system are shown by red curve for reference. The maximum BT out of the five trials is presented by gray shading. Black region indicates bounded oscillation of the stick with $|\theta| < 20$deg for 240s. Acronyms DZ, AC and JC refer to dead zone, acceleration constraint and jerk constraint, respectively.

It is observed that dead zone, acceleration limitation and jerk limitation all strongly affect the behavior of the system. In the presence of dead zone, large control gains may lead to stick falling (see Fig. 3b). While the acceleration constraint does not significantly change the region with BT of 240s (see Fig. 3c), the jerk constraint surprisingly extends the region where BT of 240s occur (see Fig. 3e). When all the three effects are involved into the model (see Fig. 3h), then the region of BT longer than 240s is significantly smaller than that of the region of linear stability. Still, there are parameter combinations, which result in BT of 240s, but the linear system for the same parameters is unstable.

In order to determine the critical length for the model, similar balance time diagrams were determined systematically for a $10 \times 10 \times 10 \times 10$ (four-dimensional) grid of the control gains $k_{p,\theta}$, $k_{d,\theta}$, $k_{p,x}$, $k_{d,x}$. If balancing with bounded oscillations was possible for any of the 10^4 combination of the control gains, then the length of the stick was decreased. The procedure was repeated until the maximum balance time out of the 10^4 trials was less than 240s. It was found that the critical length for this model is $\ell_{crit,PF} = 6$cm, which is much less than the experimentally observed critical length ($\approx$ 30cm). Note however, that this model is still lack of sensory uncertainties, imperfect actuation of

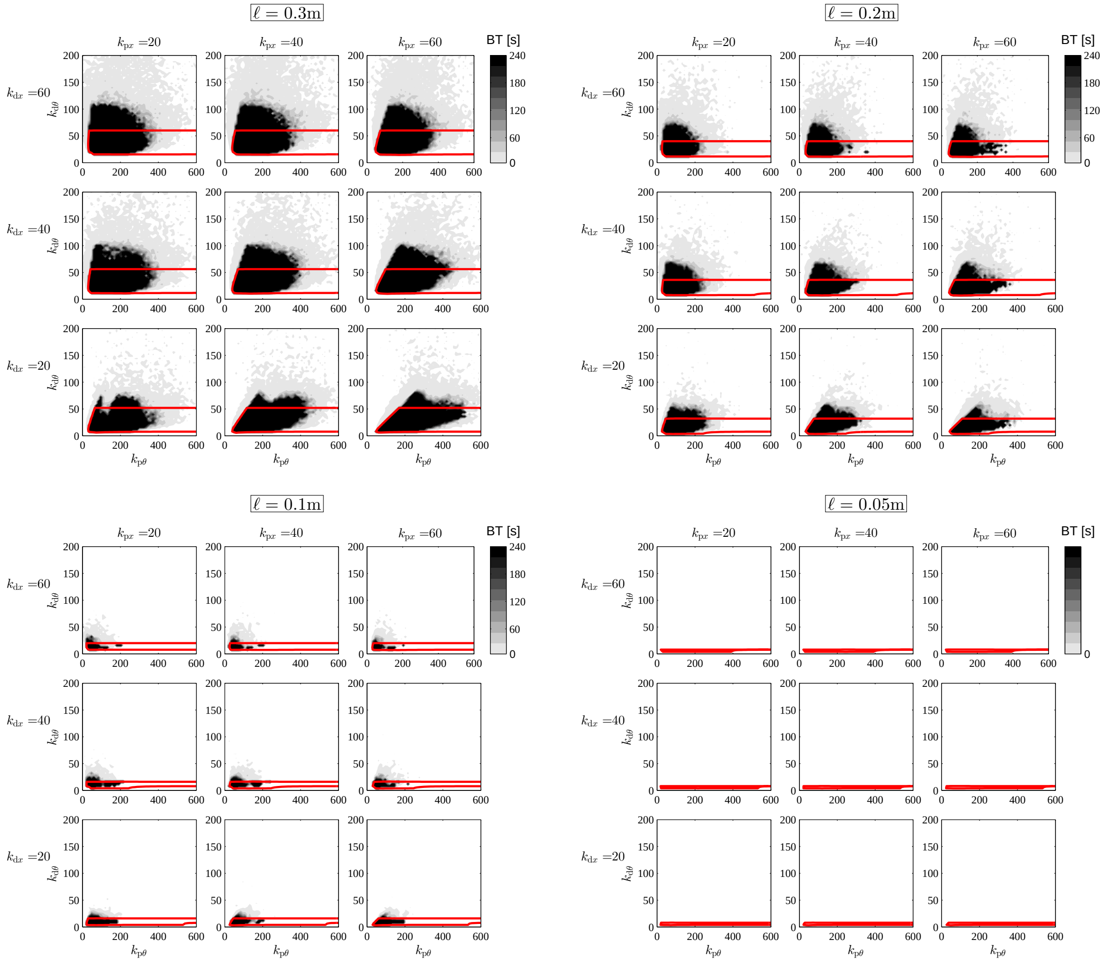

Fig. 4. Demonstraton of the decreasing balance time for decreasing stick lengths.

the control force and the internal model of the predictor feedback was also assumed to match perfectly. Consequently, the real critical length should be certainly larger.

In order to demonstrate the determination of the critical stick length for the model, a series of 3×3 balancing time diagrams is shown in Fig. 4 for different stick lengths. It can be seen that the region of parameters associated with bounded oscillations (i.e., the black-shaded region) shrinks with the decrease of the stick length ℓ and disappears between $\ell = 5$cm and $\ell = 10$cm. Note that this critical length is shorter than that observed during experimental stick balancing tests. This difference is attributed to the unmodeled imperfections of the control process performed by the nervous system during real stick balancing. Such imperfections are the inaccuracies of the sensory inputs and the motor commands and the fluctuation in the feedback delay due to the spiking communication of the neurons. We believe that implementing these imperfections into the model result in an increase of the critical length.

5. Discussion

Our observations add further support for role of a predictor feedback mechanism for expert human stick balancing. Nonlinear effects, such as sensory dead zone and control force limitation by maximum fingertip acceleration and jerk, strongly affects the dynamics of the system and the concept of stability properties cannot be used in the same way as for the linear system. Rather, the parameter regions are sought where the stick's motion is bounded. It is shown that the regions in the stability plane of control parameters where BT is at least 240s is qualitatively different from that of the linear system when a dead zone and the limitations of acceleration and jerk limitation are included into the control problem. Parameter regions that were linearly stable become unstable and prolonged BT's exist for parameter values where the system is linearly unstable. The final conclusion of the presented numerical study is that predictor feedback is able to balance sticks significantly shorter than the traditional delayed PD controllers can do so even in the presence of sensory dead zone and fingertip movement limitations. This observation strengthen the position of the predictor feedback as candidate for the mechanism of human stick balancing.

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