Periodic servo-constraints in a stick balancing problem

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Abstract

To understand the human balancing is always a scientifically challenging task. As a model of human balancing many times the problem of stick balancing is studied [1], [2]. In that studies the main focus is on the stability of the delayed controller. In contrast here the focus in on the actuation of the model. The classical model of the stick balancing problem (see: Figure 1. left) is an inverted pendulum. The classical inverted pendulum model has 2DoF but only one control forces exists thus it can be modelled as an underactuated problem. In this work a periodic control technique will be applied, which is specially developed for underactuated systems [3].

In underactuated systems the task of inverse dynamics is not well defined. Some degrees-of-freedom cannot directly be controlled, and the corresponding generalized coordinates depend on the system dynamics only. Methods that are available in the literature introduces controlled and uncontrolled generalized coordinates, and solve for the inverse dynamics with a generalized computed torque control technique. Beside the uncontrolled motion has to be calculated. This is often referred to as the internal- or passive dynamics of the system, which has to be stable to ensure the stability of the whole system. However, the stability of the internal dynamics depend on the controlled output/task. In case of the stick balancing the reducing of the inclination is the main control objective while the position of the bottom of the rod can be chosen as the internal dynamics.

As a generalization the task in the control problem is formulated by additional constraints the so-called servo constraints. In order to enhance the stability behaviour it is quite common to modify the original servo-constraints, because with this slight modification the otherwise unstable internal dynamics can be stabilized.

A different approach when the servo constraints are not modified, but periodically changed in time [3]. In one period the servo constraints are for realizing the desired motion, while in the subsequent (typically shorter) period the servo-constraints are modified to stabilize the internal dynamics. The goal of this work is to show how the periodic controller can be used in stick balancing. The sliding pendulum which is the mechanical model of the stick balancing problem is shown in (see: Figure 1. right).

The equation of motion of the swinging pendulum can be derived using the Lagrange equation of the second kind using the inclination of the rod and the horizontal position of the slider as generalized coordinates $\mathbf{q} = [x_1 \quad \vartheta]^T$,which is given in the following general form:

$$\mathbf{M}\,\mathbf{q} + \mathbf{c} = \mathbf{Q_g} + \mathbf{H}\,\mathbf{u}\,,. \tag{1}$$

Using the generalized coordinates the main task of the robot is the control of the inclination and beside this as a secondary stabilizing control objective is the regulation of the position of the slider. These control goals can be defined with servo-constraints respectively

$$\hat{\gamma} = l\,\sin(\vartheta) = 0 \quad \gamma_S = x_1 = 0\,. \tag{2}$$

Using the generalization of the computed torque control the required control force **u** could be expressed from the following equation [3]:

$$\begin{bmatrix} \mathbf{M} & \mathbf{H} \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{q} \\ -\mathbf{u} \end{bmatrix} = \begin{bmatrix} -\mathbf{c} \\ -\dot{\boldsymbol{\gamma}}_{\mathbf{q}}\dot{\mathbf{q}} - k_D\dot{\boldsymbol{\gamma}} - k_P\boldsymbol{\gamma} \end{bmatrix}, \tag{3}$$

where $\gamma = \kappa(t)\hat{\gamma} + (1-\kappa(t))\gamma_S$ is the periodic servo-constraint where $\kappa(t)$ realize the switching betwee n the two control objective.

In order to show the usability of the propoesd method a linear stability investigation was carried assuming a reflex delay. The results of the stability calculation are depicted in Figure 2 in the space of the free control gains k_D and k_P. The left hand side of Figure 2. shows the stable parameters when only the inclination of the rod is controlled. The right hand side of Figure 2. shows the stable parameters, when after each second step the original control is interrupted and the slider is controlled for one time interval.

It can be concluded that with the application of the periodic controller the control gain parameters can be selected from a wider range. Furthermore with the application of the periodic controller faster decay can achieved.

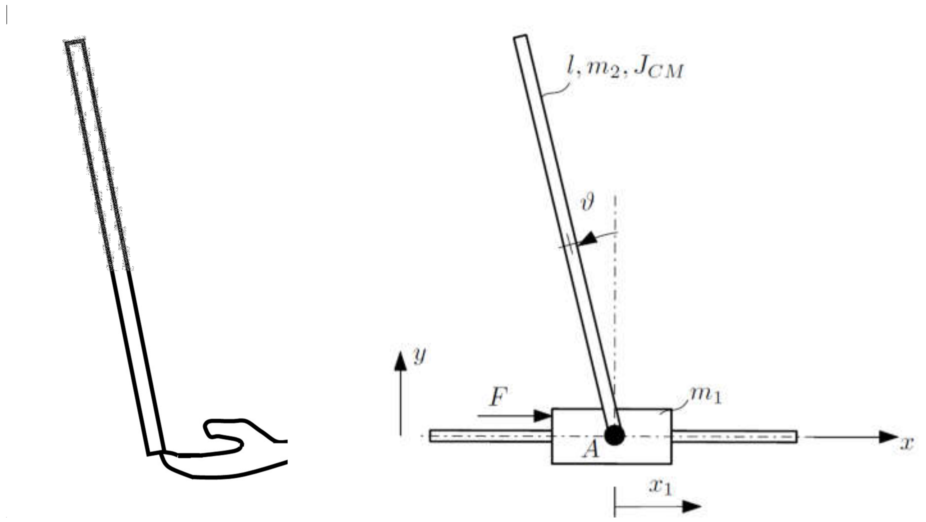

Figure 1. *left:* stick balancing problem; *right:* mechanical model of stick balancing

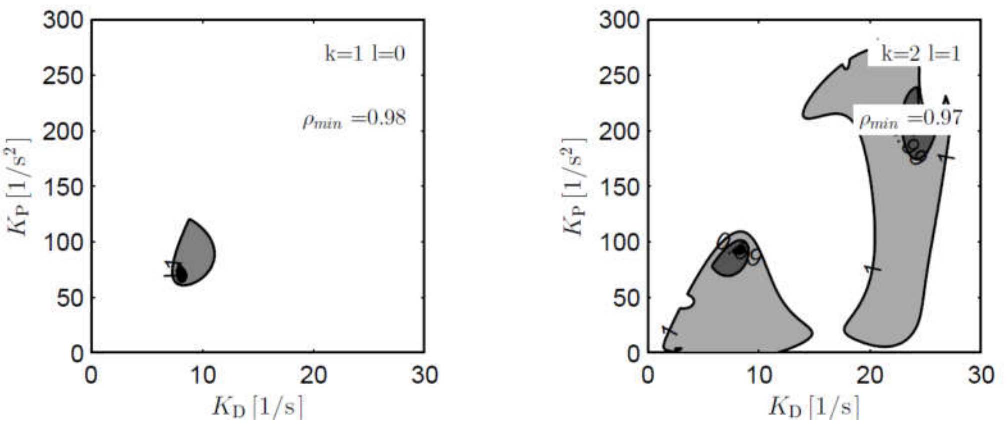

Figure 2. *left:* Stability chart of the conventional controller; *right:* Stability chart of the periodic controller

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